VII/3

$\rightarrow f(x,y) = 2x+3y$ przy ogr. $x^2+y^2=1$

$(x,y) = (\cos t, \sin t)$, $t \in [0,2\pi]$

$h(t) = f(\cos t, \sin t)$, $t\in[0,2\pi]$ ← szukamy wart. najmn. i najw.

$= 2\cos t + 3\sin t$

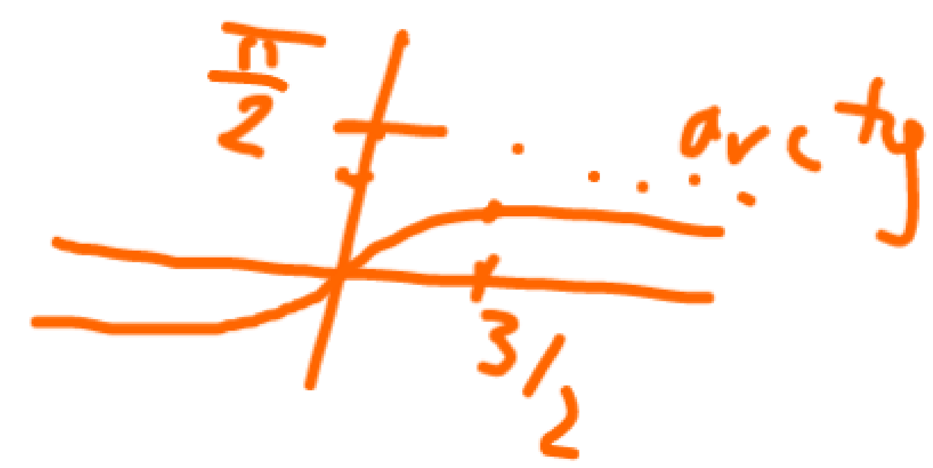
$h'(t) = -2\sin t + 3\cos t = 0$

$2\sin t = 3\cos t \,/ : \cos t$

$\cos t \neq 0$ (bo gdyby $\cos t = 0$, to $\sin t = 0$, ale $\cos^2 t + \sin^2 t = 1$)

$2\,\mathrm{tg}\, t = 3$

$\mathrm{tg}\, t = 3/2$ $\quad t_1 = \mathrm{arctg}\, 3/2 \ \vee\ t_2 = \mathrm{arctg}\, 3/2 + \pi = t_1 + \pi$



• $h(\mathrm{arctg}\, 3/2) = 2\cos(\mathrm{arctg}\, 3/2) + 3\sin(\mathrm{arctg}\, 3/2)$

$\mathrm{tg}(\underbrace{\mathrm{arctg}\, 3/2}_{t_1 \in [0, \frac{\pi}{2}]}) = 3/2$

$\frac{\sin t_1}{\cos t_1} = \frac{3}{2}$ $\qquad \cos^2 t_1 + \sin^2 t_1 = 1$

$\sin t_1 = \frac{3}{2}\cos t_1 \rightarrow \cos^2 t_1 + \frac{9}{4}\cos^2 t_1 = 1$

$\cos^2 t_1 = 4/13$

$\cos t_1 = \pm\sqrt{4/13} = +\frac{2}{\sqrt{13}}$

$\sin t_1 = \frac{3}{2}\cdot\frac{2}{\sqrt{13}} = \frac{3}{\sqrt{13}}$

$h(t_1) = 2\cdot\frac{2}{\sqrt{13}} + 3\cdot\frac{3}{\sqrt{13}} = \frac{13}{\sqrt{13}} = \sqrt{13}$

• $h(t_2) = 2\cos(t_1+\pi) + 3\sin(t_1+\pi) = -2\cos t_1 - 3\sin t_1 = -\sqrt{13}$

• $h(0) = 2\cdot 1 + 3\cdot 0 = 2$ $\qquad h(2\pi) = h(0) = 2$

Odp. najmn. wart. $-\sqrt{13}$, największa $\sqrt{13}$