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b) $\left(e^x(x^2-x+1)\right)'$

$\cancel{(fg)' = f'g'}$

$(e^x)' = e^x$

$(x^n)' = nx^{n-1}, n \in R$

$(fg)' = f'g + fg'$

$e^x(2x-1) + (e^x)' \cdot (x^2-x+1)$

$2xe^x - e^x + e^x x^2 - e^x x + e^x$

c) $\left(\dfrac{x^2+2}{x-2}\right)' = \dfrac{(x^2+2)'\cdot(x-2)-(x^2+2)(x-2)'}{(x-2)^2} =$

$= \dfrac{2x(x-2)-x^2-2}{(x-2)^2} = \dfrac{x^2-4x-2}{(x-2)^2}$

$(x^n)' = nx^{n-1}$

$\left(\dfrac{f}{g}\right)' = \dfrac{f'g-fg'}{g^2}$

$(e^x)' = e^x$

$(f(g(x)))' = f'(g(x))\,g'(x)$

$(fg)' = f'g + fg'$

d) $\left(e^{-x}\cdot(3x+1)^2\right)' = \quad -e^{-x}(3x+1)^2 + (18x+6)(e^{-x})$

$(e^{-x})' = e^{-x}\cdot(-x)' = e^{-x}(-1)$

$f(x) = e^x \quad f'(x) = e^x$

$g(x) = -x$

$\left((3x+1)^2\right)' = 2(3x+1)\cdot(3x+1)' = 6(3x+1)$

$f(x) = x^2 \quad g(x) = 3x+1$

$f(g(x))$

$f' = 2x$

e) $\ln(x^2+1)\cdot \operatorname{tg}\sqrt{x} = \underbrace{(\ln(x^2+1))'}\cdot \operatorname{tg}\sqrt{x} + \ln(x^2+1)\cdot(\operatorname{tg}\sqrt{x})'$

$(\ln x)' = \frac{1}{x}$

$(\operatorname{tg} x)' = \frac{1}{\cos^2 x} = \operatorname{tg}^2 x + 1$

$\boxed{(x^n)' = nx^{n-1}}$

$(f(g(x)))' = \underbrace{f'(g(x))}\cdot\underbrace{g'(x)}$

$(f(x)\cdot g(x))' = f'(x)g(x) + f(x)g'(x)$

$(\ln(x^2+1))' =$

$f(x) = \ln x \qquad \boxed{g(x) = x^2+1}$

$\underbrace{f'(\boxed{x}) = \frac{1}{\boxed{x}}} \qquad g'(x) = 2x$

$(\ln(x^2+1))' = \underbrace{\frac{1}{x^2+1}}_{f'(\boxed{g(x)})} \cdot \underbrace{2x}_{g'(x)}$

$(\operatorname{tg}\sqrt{x})' = \underbrace{\frac{1}{\cos^2\sqrt{x}}}_{f'(g(x))} \cdot \underbrace{\frac{1}{2}x^{-\frac{1}{2}}}_{g'(x)}$

$\sqrt{x} = x^{\frac{1}{2}}$

$\underbrace{(\sqrt{x})'} = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$

$f(x) = \operatorname{tg} x \qquad \underbrace{g(x) = \sqrt{x}}$

$f'(x) = \frac{1}{\cos^2 x}$

f)

$$\left(e^{\frac{1}{x}} \cdot \operatorname{arctg}(3-x)\right)' = \left(e^{\frac{1}{x}}\right)' \operatorname{arctg}(3-x) + e^{\frac{1}{x}} \left(\operatorname{arctg}(3-x)\right)'$$

$$(fg)' = f'g + fg'$$

$$(\operatorname{arctg} x)' = \frac{1}{1+x^2}$$

$$(e^x)' = e^x$$

$$(x^n)' = nx^{n-1}$$

$$\left(f(g(x))\right)' = f'(g(x)) \cdot g'(x)$$

$$\left(e^{\frac{1}{x}}\right)' = e^{\frac{1}{x}} \cdot \left(-1x^{-2}\right)$$

$$\left(\frac{1}{x}\right)' = (x^{-1})' = -1 \cdot x^{-2} = \frac{-1}{x^2}$$

$$\left(\operatorname{arctg}(3-x)\right)' = -\frac{1}{1+(3-x)^2}$$

$f'(g(x))$ $g'(x)$

$f(x) = \operatorname{arctg}(x)$ $g(x) = 3-x$

$f'(x) = \frac{1}{1+x^2}$

$g'(x) = 0 - 1 = -1$

$(\ln x)' = \frac{1}{x}$

$(\cos x)' = -\sin x$

$(f(g(x)))' = f'(g(x)) \cdot \underline{g'(x)}$

g) $\left(\ln(\underbrace{\cos^2 x + 1})\right)' =$

$= \frac{1}{\cos^2 x + 1} \cdot (\cos^2 x + 1)' =$

$(\cos^2 x + 1)' = (\cos^2 x)' + (1)' = (\cos^2 x)' \underset{\mathrm{I}}{=} (\cos x \cdot \cos x)' = (-\sin x) \cdot \cos x + \cos x(-\sin x) =$

$= 2\cos x(-\sin x)$

II $(\cos^2 x)' = \underbrace{2\cos x} \cdot (\cos x)' = 2\cos x\,(-\sin x)$

$f(x) = x^2$ $g(x) = \cos x$

$f'(x) = 2x$

$\boxed{(c\,f(x))' = c\,f'(x)}$

$(fg)' = f'g + fg'$

$(\arccos x)' = \frac{-1}{\sqrt{1-x^2}}$

$(\sqrt{x})' = \frac{1}{2\sqrt{x}}$

$(x^2)' = 2x$

$(f(g(x)))' = \underbrace{f'(g(x))}\cdot g'(x)$

$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$

h) $\left(\sqrt{\arccos(x^2)}\right)' =$

$= \frac{1}{2\sqrt{\arccos(x^2)}} \cdot \left(\arccos(x^2)\right)'$

$(\arccos(x^2))' = \frac{-1}{\sqrt{1-(x^4}} \cdot \underbrace{(x^2)'}_{=2x}$

(i) $\left(\frac{\sqrt{5}}{(x^2+1)^3}\right)' = \frac{\overset{0}{\overset{\shortparallel}{(\sqrt{5})'}}\cdot(x^2+1)^3 - \sqrt{5}\,((x^2+1)^3)'}{(x^2+1)^6}$

$\left\{ \underset{\underset{0}{\shortparallel}}{(\sqrt{5})'} = \frac{1}{2\sqrt{5}}\cdot(5)' = 0 \right.$

$((x^2+1)^3)' = 3(x^2+1)^2 \cdot 2x$

$f(x) = x^3 \quad g(x) = x^2+1$

$f'(x) = 3x^2$

$\sqrt{5}\cdot((x^2+1)^{-3})'$

$\left(\frac{\sqrt{5}}{(x^2+1)^3}\right)' = \left(\sqrt{5}\cdot(x^2+1)^{-3}\right)' = \sqrt{5}\cdot(-3)(x^2+1)^{-4}\cdot 2x$ ✓

$(x^{-3})' = -3x^{-4}$

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

j)

$$\left(\frac{3^{\sin^2 x}}{2^{\cos^2 x}}\right)' = \frac{\left(3^{\sin^2 x}\right)' \cdot 2^{\cos^2 x} - 3^{\sin^2 x}\cdot\left(2^{\cos^2 x}\right)'}{\left(2^{\cos^2 x}\right)^2}$$

$$a^x = \left(e^{\ln a}\right)^x = e^{x\ln a}$$

$$(a^x)' = \left(e^{x\ln a}\right)' =$$

$$= e^{x\ln a}\cdot(x\ln a)' =$$

$$= e^{x\ln a}\cdot\ln a = a^x\ln a$$

$$(a>0)$$

$$\left(3^{\sin^2 x}\right)' = \underbrace{3^{\sin^2 x}\cdot\ln 3}_{f'(g(x))}\cdot\overbrace{\underbrace{2\cdot\sin x}_{f_2'(g_2(x))}\cdot\underbrace{\cos x}_{g_2'(x)}}^{g'(x)}$$

$f(x) = 3^x$ $\qquad$ $\underline{g(x) = \sin^2 x}$

$f'(\boxed{x}) = 3^{\boxed{x}}\cdot\ln 3$

$$\left(f(g(x))\right)' = \underbrace{f'(\boxed{g(x)})}\cdot g'(x)$$

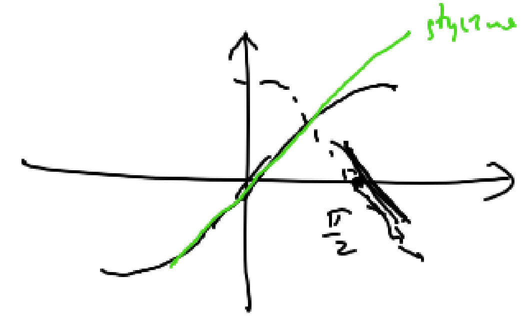
$(\sin x)' = +\cos x$

$(\cos x)' = -\sin x$

$(\sin^2 x)' =$

$f_2(x) = x^2$ $\qquad$ $g_2(x) = \sin x$

$f_2'(x) = 2x$



$$\left(2^{\cos^2 x}\right)' =$$

$$(f(g(x)))' = f'(g(x)) \cdot g'(x)$$

k) $$\left(\left(e^{-2x}+1\right)^3\right)'$$

$f(x) = x^3$ $\quad g(x) = e^{-2x}+1$

$f'(x) = 3x^2$

$$\left(\left(e^{-2x}+1\right)^3\right)' = 3\left(e^{-2x}+1\right)^2 \cdot \left(-2e^{-2x}\right) =$$

$$\left(e^{-2x}\right)' = e^{-2x} \cdot (-2x)'$$

$f_2(x) = e^x$ $\quad g_2(x) = \boxed{-2x}$

$f_2'(x) = e^{\boxed{x}}$

$0 < x < \pi$

2)

$$\left((\sin x)^x\right)' = \left(e^{x \cdot \ln(\sin x)}\right)'$$

$$f(x)^{g(x)} = \left(e^{\ln f(x)}\right)^{g(x)} = e^{g(x) \cdot \ln f(x)}$$

$$(fg)' = f'g + fg'$$

$$(\ln x)' = \frac{1}{x}$$

$$= \underbrace{e^{x \ln(\sin x)}}_{(\sin x)^x} \cdot \left(x \cdot \ln(\sin x)\right)'$$

$$\left(x \cdot \ln(\sin x)\right)' = \ln(\sin x) + x\left(\frac{1}{\sin x} \cdot \cos x\right)$$

m) $\left(\left(\arccos x+\arcsin x\right)^2\right)'=$

$=2\left(\arccos x+\arcsin x\right)\cdot\left(\frac{1}{\sqrt{1-x^2}}+\frac{-1}{\sqrt{1-x^2}}\right)=0$

$x\in(-1,1)$

$(\arccos x)'=\frac{-1}{\sqrt{1-x^2}}$

$(\arcsin x)'=\frac{1}{\sqrt{1-x^2}}$

$(x^2)'=2x$

$\left(f(g(x))\right)'=f'(g(x))\cdot g'(x)$

$(f+g)'=f'+g'$

$\Rightarrow \left(\arccos x+\arcsin x\right)^2=\text{konst.}$ he $(-1,1)$

$\arccos 0+\arcsin 0=\frac{\pi}{2}+0=\frac{\pi}{2}$

$(x^{-1})'=-1\cdot x^{-2}=\frac{-1}{x^2}$

$\Rightarrow \arccos x+\arcsin x=\frac{\pi}{2}$ die $x\in(-1,1)$

n) $\left(\underbrace{\ln(2x)}_{\ln 2+\ln x}+\underbrace{\ln\frac{3}{x}}_{\ln 3-\ln x}\right)'=\frac{1}{2x}\cdot 2+\frac{1}{\frac{3}{x}}\cdot\left(3\cdot x^{-1}\right)'=\frac{1}{x}+\frac{x}{3}\,3\left(-\frac{1}{x^2}\right)$

$(\arcsin x)'=\frac{1}{\sqrt{1-x^2}}\cdot 1$

$=\frac{1}{x}-\frac{1}{x}=0$

o)

$$\left(\frac{\ln 2021}{x^2+1}\right)' = \ln 2021 \cdot \left(-\frac{1}{(x^2+1)^2}\right) \cdot (x^2+1)' =$$

$$= \frac{-\ln 2021 \cdot 2x}{(x^2+1)^2}$$

$$\cancel{\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}}$$

$$\left(\frac{1}{x}\right)' = -\frac{1}{x^2}$$

$$(fg)' = f'g + fg'$$

p)

$$\left(\underbrace{e^5 \sin(2x)} + \sqrt{\pi} \cos(3x)\right)' = e^5 \cos(2x)\, 2 + \frac{1}{2} \pi^{-\frac{1}{2}}\, \square +$$

$$+ \sqrt{\pi} \cdot (-\sin(3x))\, 3$$