

1. $f = x^3 - 3xy^2$. Oblicz: $f_{xx} + f_{yy}$.
2. $f = xy(x^2 - y^2)$. Oblicz: $f_{xx} + f_{yy}$.
3. $f = \ln(x^2 + y^2)$. Oblicz: $f_{xx} + f_{yy}$.
4. $f = \operatorname{artg} \frac{x+y}{1-xy}$. Oblicz: f_{xy} .
5. $f = \operatorname{artg} \frac{y}{x}$. Oblicz: $xf_x + yf_y$, $xf_y - yf_x$, $f_{xx} + f_{yy}$.

Znajdź ekstrema lokalne funkcji:

$$\begin{aligned} f &= 5x^2 + 2y^2 - 6xy - 2x, \quad f = y^3 + 3x^2y - 3xy, \quad f = x^3 + 3xy^2 - 15x - 12y, \\ f &= xy(x + y - 3), \quad f = (x + y + 1)^3 - 27xy, \quad f = (x + y)^3 - 3x^2 - 3y^2, \\ f &= x^3 + y^3 - 3xy, \quad f = x^3 + 3xy^2 - 15x - 12y, \quad f = x^2 + y^2 + \frac{1}{x + y}, \\ f &= x + y + \frac{1}{xy}, \quad f = xy + \frac{1}{x} + \frac{1}{y}, \quad f = 2(x + y)^2 + \frac{1}{x} + \frac{1}{y}. \end{aligned}$$