EMS School on Industrial Mathematics
Bedlewo, 11–18 October, 2010

Project: *Problem with air bubbles in sausage production process*

Report

Joachim Linn, Włodzimierz Bąk, Jacek Domagała, Krystian Kazaniecki, Michał Kawiak, Witold Kosiński, Andrzej Palczewski, Bogna Pawłowska

Executive Summary

Having considered various possible reasons for bubbles appearance, we chose the most probable hypothesis and investigated it in-depth. We suppose that air bubbles appear at the sausage skin due to high pressure exerted by intestine on the machine's steel tube. There is always some air leftover in the harmonica, but when the pressure is high, it cannot escape. There are three main factors for high pressure: small radius of the intestine used, its little elasticity and natural irregularities of its surface. We have provided a quantitative mathematical model to describe pressure as a function of several adjustable factors.

Introduction

The Objectives

The company producing natural sausages would like to rise the profit of its production by increasing the quality of its product. In this process they have discovered that in a small fraction of their product air bubbles appear just under the sausages skin. The objective of the project is to find mechanism of bubble formation and propose an improvement which the appearance of bubbles can prevent altogether.

Due to the company secrets the information available is very limited. Fortunately additional data can be collected from the sausage filling machine manufacturer (Handtmann) and by performing some approximate calculations. It should be emphasized that the appearance of air bubbles has a random character and only about 1% of production has visible bubbles.

The Goals

- recognize the mechanism which produces air bubbles,
- give a quantitative solutions of bubbles formation,
- propose the improvement of sausages quality without increasing the cost of production,
- propose further research on that problem.

Description of the process

In the sausage filling process a vacuum filling machine produced by Handtmann is used (a visualisation of the production process can be seen on a video provided on the Handtmann web page [1]). Handtmann is a worldwide supplier to the meat processing trade and meat industry. Handtmann manufactures filling and portioning machines for all sizes of business. It offers vacuum filling machine from small and medium-scale operations to industrial machines. As Handtmann claims ”Product quality is always first class, irrespective of whether the product is portioned into artificial, collagen or natural casings”. Nevertheless our customer has some problems with portioning into natural casing where for a small fraction of production small air bubbles appear.

Our customer is a medium-size company. It produces different type of sausages. The sausage in problem is a traditional sausage in which homogeneous stuffing is mixed with meat containing large pieces.

The sausage mass consists of: 30% of homogeneous stuffing with diameter of particle 0.5 mm, in temperature 8°C and 70% of large meat pieces with diameter of particle 8 mm, in temperature 3–4°C. This mass is mixed for 5 minutes (3500 turns/min) in vacuum conditions.

Next the meat mass is left for about 4 hours before putting into filling machine. The filling machine works also in the vacuum conditions (98% vacuum). The maximal velocity of filling is 50 Liters/min. The pressure of filling is till 80 bars.

The filling is done with a 22 mm filling tube. The casing which is a natural intestine is put on the filling tube with a spooling device with a large speed (a natural intestine is 10 m long and it is spooled in few seconds). The tube has a length of 0.4 m and the wrapped intestine occupies 10–20 cm of the tube.

Before filling the intestine are kept for 10 minutes in salty water of unknown temperature. At the manufacturer of the fillinge machine website there is an information that intestines should be wet for 2 hours in water at 40°C.

During the sausage production the intestine is filled with the meat mass with a speed of 0.5 m/s (the whole 10 m of intestine is filled in 20 sec).

The produced sausage has a diameter of 24–26 mm.

When the air bubbles appear they have a size of 3–5 mm and are of a lens shape. The bubbles are distributed irregularly over the whole length of the sausage (whole 10 m). On the other hand it happens very irregularly – sometime just one sausage, sometime several consecutive sausages in a row. But the overall bad product is of order o 1% of the whole production.

Hypothesis

A number of hypotheses was formulated to explain the formation of air bubbles. Below are those hypothesis, which explain the most probable mechanism of the process:

- Air bubbles appear in the intestine of a smaller diameter. The main factor in deciding whether the bubbles will be visible or not is pressure exerted by the intestine on a steel tube of filling machine.
- Torus-like air bubbles are hidden in the folds of a "harmonica." At the time of the stretch bending, in accordance with the principle of least action torus-shaped bubble changes to "lens-shaped" one which is located at random place.
- The pressure exerted by the gut depends in a linear manner from the Young's modulus. We suspect that through the process of preparation (hydration in salty water) the value of the Yuang's modulus can be lowered by two orders of magnitude.
- During the process of dragging an intestine at high speed, air gets between the bowel and the tube of the filling machine. This process is largely independent of the earlier preparations of the products, so we do not have a significant influence on the amount of air contained in the "harmonica".

The main factor which has to be taken into account is the fact that bubbles appear only in 1% of products. This raises the question why in 99% of cases air bubbles manage to escape through the open end of the gut?

The most plausible explanation of this last observation is that for the initial radius of the intestine (less than a certain r_ϵ, with certain Young's modulus determined by the preparation process) the thrust force by which the intestine acts on the steel tube prevents escaping air closed in bubbles through the open end. For larger intestines the force is too small to trap air bubbles inside the intestine.

On the other hand, if the initial radius of the intestine is the main factor, why the operators of the filling machine did not report this fact.

The only explanation is the fact the critical value of the radius is in the area where the operator of the machine cannot fill manually a difference in thrust force. If only 1% of production creates problems it means that the critical radius is very far away from a mean value of the radius.

Further clarification

To establish the mechanism of formation and transport of air bubbles during the manufacture of sausage a number of other possibilities has been considered.

First, the following facts have established:

- Meat is prepared at low pressure (150-200 Pa, which corresponds to 1.5% - 2%) of atmospheric pressure
- It is stored in atmospheric pressure for about 4 hours
- Filling machine works again at low pressure (150-200 Pa).

These facts clearly show that it is reasonable to assume that the volume of air in the meat mass is so small that we can ignore it.

Then the only plausible assumption would be that the air gets inside with the intestine, when it moves along the steel tube filling machine.

Another important point is the answer to the question why the bubbles appear in only 1% of cases. This raises the suspicion that the mechanism responsible for the formation of bubbles is parameter specific for the intestine but not for the meat mass, chemicals used for cleaning equipment or the device itself.

It seems that the critical parameter is the size (radius) of an intestine in unstretched condition.

More detailed explanation of the hypothesis:

Membrane (intestine) is stretched at high speed (0.5 m / s) on the tube filling machine, where it curls up in a "harmonica". Between the tube and the harmonica is a certain volume of air. Initially, both ends of the bowel are open, but after the start of the filling machine only the rear end (the end of the remainder of the tube) remains open. If the contact force exerted by the membrane is sufficiently small, air can easily escape from the space between the membrane and a tube through the open end. If the force is large enough, some air remains in the space between the membrane and the tube, causing the occurrence of bubbles in the final product. An important factor influencing the incidence of bubbles is a velocity of the movement along the tube. If this rate was lower, the air bubbles would more likely escape through the open end.

When you move the gut on the tube there creates a vacuum between the membrane and tube. This difference, however, is 125 Pa (corresponding to 1.2% of atmospheric pressure). Given this result, we found that depression has insignificant effect on the entire process.

Obviously there is also possibility that during the production micro-bubbles are formed and transported with a layer of transported water (to moisten the steel tubes). Their size is quite comparable to the width of the water layer, which we can estimate by 10 microns. taking under consideration this fact, for forming a noticeable-sized bubble, there would have to collect 10^6 follicles in less than $0.8s$ (so a particular point placed of the shell moves along the tube). It is clear that this mechanism also has a negligible effect on the formation of bubbles.

Therefore the main factor (affecting more than 99%) is the pressure exerted by the gut on the walls of the device. The pressure that exerts stretched gut on a tube have been calculated and it was found that it is a determining factor whether bubbles appear or not.

Pressure of the intestine membrane

Table 1 contains data describing the process of sausage production.

parameter	value	description	confidence
r_0	10 mm	radius of intestine in natural conditions	our guess
r_1	12 mm	radius of filled (stretched) intestine	factory value
h	0.4 m	length of filling tube	factory value
L	10 m	length of intestine in natural conditions	factory value
l	10–11 m	length of filled (stretched) intestine	our guess
v	0.5 m/s	speed of filling intestine	factory value
E	0.01–1 GPa	Young's modulus of intestine	our guess
η	200	dynamic viscosity of meat mass	our estimate

Table 1: Sausage production data

The pressure equation is the following one

$$p(r_1) = \frac{1}{\beta}\frac{E}{2(1-\nu^2)}\frac{t_0}{r_0}[(\alpha^2-1)+\nu(\beta^2-1)], \tag{1}$$

where $\alpha = \frac{r_1}{r_0}$, $\beta = \frac{l}{L}$, $\nu = 0.5$ and $t_0 = 0.0001\,m$.

For our model we can assume that $\alpha = [1 \ldots 1.2]$ and $\beta = [1 \ldots 1.1]$.

To defend the choice of Young's modulus for intestine we have colected data available in internet [2, 3]. These data are in Table 2.

material	Young's modulus in GPa
ABS plastics	2.3
Acrylic	3.2
Collagen	**3.7–4**
Rubber	**0.01–0.1**
Skin (human)	**0.01**
Nylon	2–4
Polycarbonate	2.6
Polyethylene HDPE	0.8
Polyethylene PET	2–2.7
Polypropylene	1.5–2

Table 2: Young's modulus of different materials

Hence the Young's modulus for intestine is assumed between the values for skin and collagen.

Derivation of a simple model to estimate the normal pressure of stretched natural pig intestine on the steel tube of a sausage filling machine

The principle idea of our model is the following: We treat the intestine as a cylindrical elastic membrane. We have to consider moderate geometrically nonlinear effects in longitudinal stretching and radial extension, with deformation ratios of up to 1.2 (i.e. about 20%). We choose *Koiter's membrane shell model* with linear elastic, homogeneous and isotropic material behavior as an appropriate mechanic model of the intestine membrane structure, cf. [5] and [6].

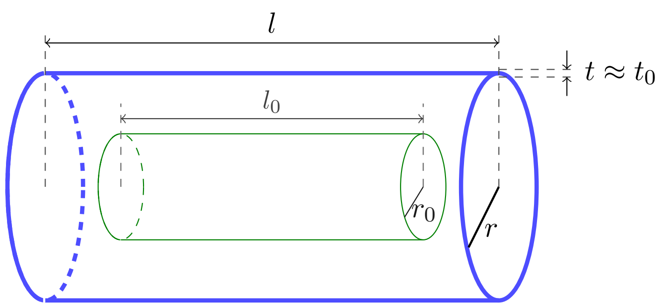
The starting point of our derivation is the formula for the total elastic energy W_{shell} of the deformed shell, which has roughly the following structure:

$$W_{shell} = \frac{Z}{2} \cdot \big(\text{change of metric}\big)^2 + \frac{D}{2} \cdot \big(\text{change of curvature}\big)^2.$$

Here

$$Z = \frac{Et_0}{(1-\nu^2)} \qquad \text{and} \qquad D = \frac{Et_0^3}{12(1-\nu^2)}$$

are the membrane and bending stiffness parameters of the shell, which depend on Young's modulus E (units: $Pa = N/m^2$) and the dimensionless Poisson number ν of the shell material as well as the initial thickness t_0 of the undeformed shell. This model is appropriate for moderate geometrical nonlinearities, but not too large local strains. It contains fully nonlinear strain measures (i.e. the metric and curvature tensor of the shell surface), but the material behavior is assumed to be linear. This is indicated by the appearance of the stiffness constants Z and D which are well known from linear plate and shell models.



Introducing a typical tube radius of $r \approx 10$ mm and the typical thickness of $t_0 \approx 0{,}1$ mm, a rescaling of the energy shows that the membrane part of the energy is proportional to $\frac{t_0}{r} \approx \frac{1}{100}$, while the bending part is proportional to the third power $\left(\frac{t_0}{r}\right)^3 \approx 10^{-6}$ of this parameter. Therefore the contributions of the bending term relative to the membrane term is negligible for the computation of the total energy of the deformed shell in the special deformation case considered by us.

The initial shape of the tube is a cylinder of the length l_0, radius r_0 and thickness t_0. In the deformed state the tube has different parameters: $l > l_0$, $r > r_0$ and $t \leq t_0$. In our model we have:

$$\frac{l}{l_0} \in [1,\ 1.2], \qquad \frac{r}{r_0} \in [1,\ 1.2].$$

As the strains are not too large, we may safely assume — and do so — that the influence of the reduction of the shell thickness is small (i.e. $t \approx t_0$), for the following reason: For radial and longitudinal strains in the range given above, the thickness can reduce up to factor $\frac{2}{3}$ in the case of a fully incompressible material ($\nu = 0.5$). If we take the variation of the thickness due to stretching and radial expansion into account during the derivation of our final pressure formula, the normal pressure will be reduced by an additional term of order $(t_0/r_1)^2 \approx 10^{-4}$, which we can safely omit as a second order effect.

Using basic notions from the classical differential geometry of surfaces one can obtain the deformation gradient and the change of the metric tensor of the membrane surface corresponding to the homogeneous deformation of our cylindrical tube. Inserting these ingredients into Koiter's energy formula we obtain the following expression for the elastic energy of the deformed cylindrical membrane:

$$W_{shell} = 2\pi r_0 l_0 \cdot \frac{Et_0}{8(1-\nu^2)} \left\{ \left(\frac{r^2}{r_0^2} - 1\right)^2 + \left(\frac{l^2}{l_0^2} - 1\right)^2 + 2\nu \left(\frac{r^2}{r_0^2} - 1\right)\left(\frac{l^2}{l_0^2} - 1\right) \right\}.$$

The normal pressure $p(r)$ to be applied inside the membrane surface or radius $r \geq r_0$ and length $l \geq l_0$ to equilibrate the elastic membrane forces can be calculated by computing the total work which has to be done during a radial inflation of the membrane at constant stretching:

$$W_{shell}(r,l) = \int_{r_0}^{r} p(r) \cdot 2\pi r l \, dr \qquad \Longrightarrow \qquad p(r) = \frac{1}{2\pi r l} \cdot \frac{\partial W_{shell}}{\partial r}.$$

In our case the final radius r_1 is known. Introducing the stretching ratios $\alpha = \frac{r}{r_0}$ and $\beta = \frac{l}{l_0}$ we can express the above formulas as follows:

$$p(r_1) = \frac{E}{2(1-\nu^2)} \cdot \frac{t_0}{r_1} \cdot \frac{\alpha}{\beta} \left[(\alpha^2 - 1) + \nu(\beta^2 - 1) \right].$$

This formula gives a rough estimate of the normal pressure of the stretched and expanded intestine on the steel-tube.

To obtain the order of magnitude of this pressure we need to estimate the Young's modulus E of pig intestine. This material parameter presumably varies between typical values for human skin ($E = 10$ MPa) and collagen fibres $E = 1$ GPa, which differ by two orders of magnitude. An intermediate value of $E \approx 0.1$ GPa might be a reasonable choice for approximations. Typical value of the Poisson's ratio for organic intestine material may be close to that of rubber $\nu = 0.5$ (see [4]). Taking the pre-factor into $\frac{t_0}{r_1} \approx 10^{-2}$ account this yields

$$p(r_1) \approx \frac{E}{150} \cdot \frac{\alpha}{\beta} \left[\alpha^2 + \frac{1}{2}\beta^2 - \frac{3}{2} \right].$$

If we take the final radius $r_1 \approx 10$ mm as a fixed given quantity, the latter formula may be used to analyze the normal pressure of the intestine on the steel-tube as a function of the variation of the initial radius r_0 — this is quite hard to estimate — of the intestine and the stretching of the intestine on the steel tube.

If air bubbles are present between the steel-tube and the intestine, they are probably surrounded by water. The pressure $p(r_1)$ can then be used to estimate the pressure inside the air bubbles according to Laplace formula

$$p_{bubble} = p(r_1) + \text{(surface tension)} \times \text{(mean bubble curvature)}.$$

However, a rough estimate for typical (visually observable) bubble sizes shows that the additional pressure term proportional to the surface tension between air and water is of the order of 100 Pa and therefore also can be neglected. So we may assume that $p_{bubble} \approx p(r_1)$ holds in our estimation of the pressure of such air bubbles.

According to our considerations, high pressure increases a probability of appearance of bubbles. At some critical value (we suppose 1,5–2 bars) this probability substantially increases. But obviously there is a possibility, that despite really unfavorable conditions the air bubbles will not appear. On the other hand, there is also a possibility, that despite favorable conditions, air bubbles appear. Our solution leads to lower the probability as much as possible. But we cannot guarantee that the resulting calculations will explain the bubbles behavior in every single case.

Tables and graphs describing pressure dependence on α, β and Young's Modulus

The following section describes the pressure dependence of the radial stretch ratio α, the longitudinal stretch ratio β and Young's modulus. In the tables we can see how the pressure change depends on the Young's modulus and α for a fixed parameter β. These dependencies seem to be particularly important since α is a factor that has a large influence on the pressure, and the Young's modulus is a parameter which could be controlled by a suitable process of preparation of intestines. As we expect air bubbles should appear at high pressure (most likely more than 2 bars), which corresponds to the lower right corners of the tables.

$\alpha \backslash E$	10	50	100	500	1000
1	0	0	0	0	0
1,05	0,057	0,285	0,569	2,847	5,694
1,1	0,117	0,583	1,167	5,833	11,667
1,15	0,179	0,896	1,792	8,958	17,917
1,2	0,244	1,222	2,444	12,222	24,444

Table 3: Pressure [in bar] exerted on the membrane at a fixed parameter $\beta = 1$

$\alpha \backslash E$	10	50	100	500	1000
1	0,027	0,136	0,271	1,356	2,712
1,05	0,081	0,407	0,813	4,067	8,135
1,1	0,139	0,691	1,382	6,911	13,823
1,15	0,198	0,989	1,978	9,888	19,775
1,2	0,26	1,3	2,6	12,996	25,992

Table 4: Pressure [in bar] exerted on the membrane at a fixed parameter $\beta = 1,05$

$\alpha \backslash E$	10	50	100	500	1000
1	0,053	0,265	0,53	2,652	5,303
1,05	0,105	0,524	1,048	5,24	10,48
1,1	0,159	0,795	1,591	7,955	15,909
1,15	0,216	1,079	2,159	10,795	21,591
1,2	0,275	1,376	2,753	13,763	27,525

Table 5: Pressure [in bar] exerted on the membrane at a fixed parameter $\beta = 1,1$

In the following graph we present the relationship between force (pressure) exerted by the gut on steel tube and 3 parameters: the Young's Modulus (E), coefficient of radial stretch (α) and the longitudinal stretch ratio (β). The Young's modulus is set at level of 100 MPa. The coefficients α and β vary between 1 to 2. The right graph shows an enlarged part of the left part (the color changed to improve readability). The values of α and β vary in the range of 1 to 1.2. We believe that these values correspond to the actual production conditions, which explains the importance of this part of the chart.

We suspect that the critical pressure, which may cause the occurrence of bubbles is about 2 bars (light-blue color on the right graph).

In the model there is a linear dependence on Young's modulus. If the value measured experimentally proved to be significantly different from the assumed (100 MPa), the pressure read from the chart would have to be scaled by a constant factor (measured value)/100.

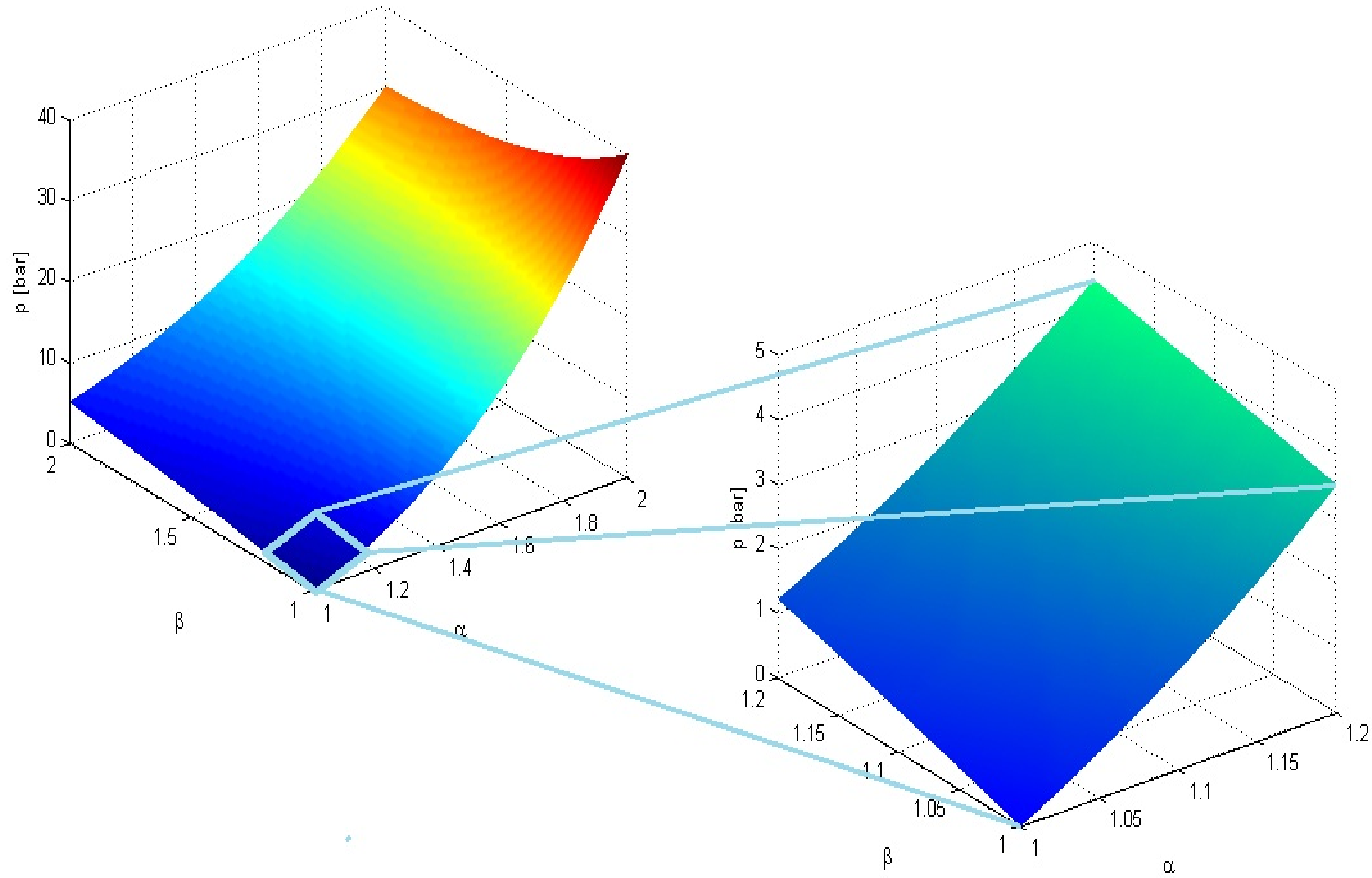


Figure 1: Graph showing pressure dependence of α and β, On the left general graph, on the right the values more specific for our model

In the next two graphs we show the dependence of pressure exerted by the intestine from the Young's modulus (E) (from 10 to 1000 MPa) and the radial stretch ratio α. Longitudinal extention ratio was set at 1.02 and 1.1 respectively.

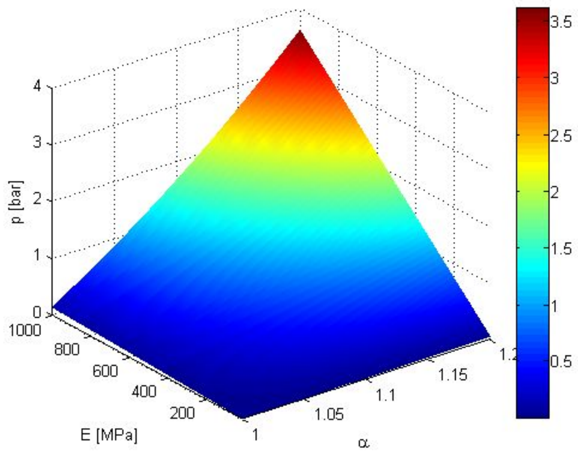


Figure 2: $p(\alpha, E)$ with $\beta = 1,02$

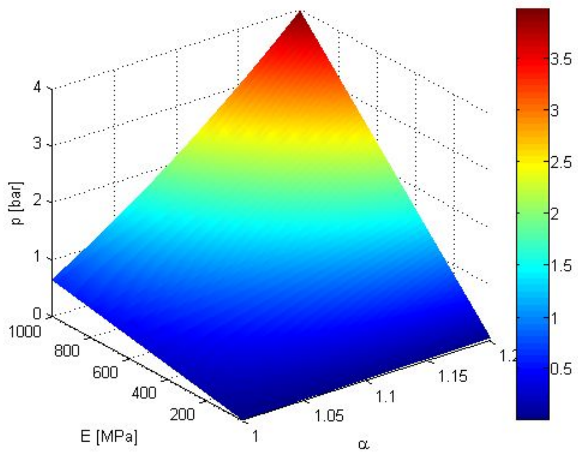


Figure 3: $p(\alpha, E)$ with $\beta = 1,1$

We see that if we compare the values of these two graphs for some α and E (especially for $\alpha \approx 1.2$), the difference does not exceed 10%. Thus we see that the growth of α has much more significant impact on the growth pressure.

References

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[5] P. Ciarlet –An Introduction to Differential Geometry with Applications to Elasticity, *J. Elasticity*, **78–79** (2005), 1–215.

[6] A. H. Corneliussen and R.T. Shield – Finite Deformation of Elastic Membranes with Application to the Stability of an Inflated and Extended Tube, *Arch. Rational Mech. Anal.*, **7** (1961), 274–304.

Appendix

Considered mechanisms of bubbles formation

While trying to find a suitable mechanism many hypotheses have been raised. Most of them we managed to reject for various reasons, but for the documentation purposes we present them in this appendix.

1. When mixing the meat mass air gets to it in the form of micro-bubbles, which, after filling join together and diffuse into the outer layers.

   Counterarguments:

   - Both mixing and filling takes place at low pressure (150-200 Pa), so the amount of air trapped in the meat mass is negligible.
   - Bubbles are visible immediately after filling, the diffusion cannot last fractions of seconds.
   - It would be impossible to determine why the bubbles appear in only 1% of products.

2. Bubbles in fact contain no air, but other gas which is formed in a chemical reaction of meat mass with the intestine, or diffusing from the inside of meat mass.

   Counterarguments:

   - Bubbles are visible immediately after filling, the diffusion takes time.
   - It would be impossible to determine why the bubbles appear in only 1% of products.
   - If intestine would react with a mass of meat, the process of production would probably not have been approved by the health control.

3. Visible bubbles form by merger of micro-bubbles transported in a layer of water between an intestine and a steel tube.

   Counterarguments:

   - Bubbles would have to be comparable to the thickness of layer of water, which we can estimate by 10 microns.
   - there would be necessary to connect about 10^6 micro-bubbles to form a noticeable bubble.
   - It would be impossible to determine why the bubbles appear in only 1% of products.

4. Bubbles appears in intestines, whose radius is bigger then radius of the tube of filling machine. While filling, some bubbles from outside the intestine are sucked by underpressure created by movement of the bowel along the tube.

   Counterarguments:

   - We assume, that a worker keeps his hand on the tube. In this case air bubbles would be easily squeezed.
   - Moreover, on a machine producer's webpage we saw a special ring-shaped device around the tube. We know, that the air bubbles problem appears around the world. If this was a mechanism, it would be already solved. That is why we excluded such possibility.
   - Pressure of air trapped under the gut is equal to 99% of atmospheric pressure. The difference is to small to have a noticeable influence on the process.

- Taking this result under consideration we cannot explain the mechanism of possible sucking the air bubbles.

5. Bubbles already trapped under intestine are sucked to the prepared sausage by the underpressure.

   Counterarguments:

   - Pressure of air trapped under the gut is equal to 99% of atmospheric pressure. The difference is to small to have a noticeable influence on the process.
   - In intestines of typical radius, that would be easy to squeeze the bubbles out.
   - This effect in fact can constitute a minor effect, but as we estimated bubbles formed by this effect will have size of $10^{-8}m = 10nm$, which will make them invisible.

6. Bubbles appear if air was not squeezed before stretching the bowel on the tube.

   Counterarguments:

   - Air appears always between gut and steel tube. This proves, that there must be another mechanism of forming air bubbles.
   - We could not explain why bubbles appear only in 1% percent of products.

7. Bubbles appear because of defect of the filling machine

   Counterarguments:

   - Bubbles in sausages is a worldwide known problem.
   - In this case we would have to assume defect of almost every filling machine.