Collision Avoidance Strategies for a Three Player Game

Sriram
Technology Philanthropist,
Schenectady,
NY
e-mail: sshankaran@gmail.com

Dušan Stipanović
Assistant Professor,
Department of Industrial and Enterprise Systems Engineering, and Coordinated Science Laboratory,
University of Illinois at Urbana-Champaign
e-mail: dusan@uiuc.edu

Claire Tomlin
Professor,
Department of Electrical Engineering and Computer Science,
University of California, Berkeley
e-mail: tomlin@eecs.berkeley.edu

Abstract

Collision avoidance strategies for a game with three players, two pursuers and one evader, are constructed by determining the semi-permeable curves that form the barrier. The vehicles are assumed to have the same capabilities, speed and turn-rates. The game is assumed to be played on a two-dimensional plane. The usable part and its boundary are first determined along with the strategy along the boundary. Semi-Permeable curves are evolved from the boundary and closure of the barrier is determined from the condition that leads to intersection of two semi-permeable curves. As in the game of two cars, universal curves and the characteristics that terminate and eminate from the universal curve are used to fill voids on the barrier surface. This study benefits from the following published work on barriers for differential games, Isaacs [1], Merz [2, 3], Pachter and Getz [4], Bardi [5], Falcone and Bardi [6], Patsko and Turova [7] and Mitchell, Bayen and Tomlin [8].

Keywords : Differential Games, Dynamic Programming, Numerical Methods, Game Theory.

1 Dynamics and RPEs

The dynamics of the vehicles are assumed to be

$$\frac{dx_i}{dt} = W\cos(\phi_i); \quad \frac{dy_i}{dt} = W\sin(\phi_i); \quad \frac{d\phi_i}{dt} = \sigma_i \tag{1}$$

$1 \leq \sigma \leq 1$ is the control input that the vehicles choose and W is assumed to be 1. Capture is defined by the time taken by one of the pursuers to reach a radius, l, of the evader. The two pursuers are referred to with subscripts 1 and 2 and the evader is labeled with a subscript e. To reduce the dimensionality of the problem, the dynamics is written in relative coordinates with the evader fixed at the origin as

$$\frac{dx_1}{dt} = -\sigma_e y_1 + \sin(\phi_1); \quad \frac{dy_1}{dt} = -1 + \sigma_e x_1 + \cos(\phi_1); \quad \frac{d\phi_1}{dt} = -\sigma_e + \sigma_{p1} \tag{2}$$

$$\frac{dx_2}{dt} = -\sigma_e y_2 + \sin(\phi_2); \quad \frac{dy_2}{dt} = -1 + \sigma_e x_2 + \cos(\phi_2); \quad \frac{d\phi_2}{dt} = -\sigma_e + \sigma_{p2} \tag{3}$$

The pay-off for this game is of the following type,

$$J = \max_e \ \min_p \left(\int_0^{t_1} 1dt, \int_0^{t_2} 1dt \right)$$

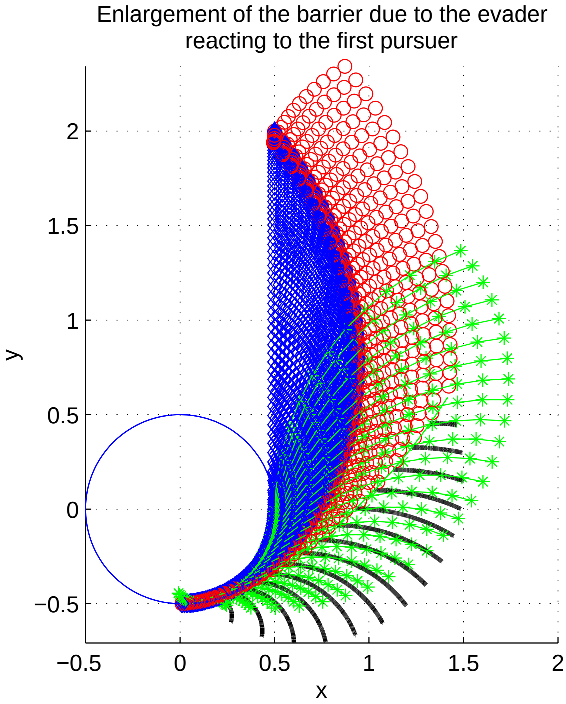

Figure 1: Enlargement of Right Barrier projected onto the (x, y) plane for relative headings ranging from $-\pi$ to 0 (the boundary starting at $(0.5, 0)$ corresponds to $-\pi$ and the boundary starting at $(0, -0.5)$ corresponds to 0), blue points lie on the portion that leads to the capture circle with the evader and the second pursuer turning in opposite directions but the evader evading the first pursuer, the red points lie on portion of the barrier that leads to the universal curve and reach the capture circle, green points are portions of the right barrier that lead to the capture circle with the evader and the second pursuer turning in the same direction, black curves lead to the universal curve with the evader and the second pursuer turning in the same direction and then reach the capture circle. All trajectories graze the capture circle and hence are collison avoidance trajectories for capture circle radius less than 0.5.

The value of the game, V, is the minimum of the minimum time-to-reach function for each of the pursuers $\mathcal{T}_i$,

$$\mathcal{T}_i(x) = \min\{t_i : x(t_i) \in \mathcal{C}\}; \quad i = 1, 2 \tag{4}$$

where t_i is the time it takes for the i^{th} pursuer to reach the capture circle. The value function for the game is given by,

$$V = \min(\mathcal{T}_i); \quad i = 1, 2 \tag{5}$$

where $\mathcal{C}$ is a circle for each relative heading. As the pay-off is of the nondifferentiable type, it is not easy to adapt Isaac's approach for the game of degree but we are currently only concerned with collison avoidance strategies which are derived from the following definition of the barrier surface

$$\min_{\sigma_{p1}} \min_{\sigma_{p2}} \max_{\sigma_e} < \nabla\nu \cdot f >= 0$$

2 Conclusions

Preliminary results indicate that it is possible to systematically construct the barrier for games with more than two players, starting with the barrier for the two player game. The final version of the paper will provide more details of the calculation along with a description of the increase in the left barrier for the game of three cars.

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