ALGEBRA Homework List 1.

Analytic geometry on the plane

1. Straight line ℓ and a point M on the plane are given. Write the parametric and the normal forms of equation for a line which contains M and is

(1) parallel to ℓ ; (2) orthogonal to ℓ

in the following cases:

- a) $M(2, -3), \quad \ell : 2x 3y + 5 = 0;$
- b) $M(1, -2), \quad \ell : 5x y + 3 = 0;$
- c) $M(4, -1), \quad \ell : -3x + y + 2 = 0.$

2. Straight line ℓ and a point *P* on the plane are given. Find the point *Q*, which is the projection of *P* on ℓ , and the point *R*, which is symmetric to *P* w.r.t. ℓ

- a) $P(-6,4), \quad \ell: 4x 5y + 3 = 0;$
- b) $P(-5, 13), \quad \ell : 2x 3y 3 = 0;$
- c) P(-8, 12), ℓ contains $M_1(2, -3), M_2(-5, 1)$;
- d) P(8,-9), ℓ contains $M_1(3,-4), M_2(-1,-2)$.

3. For the triangle ABC with A(-2,3), B(4,1), C(6,-5), write the parametric and the normal forms of equation for a line which contains

- a) the median;
- b) the bisector;
- c) the altitude

containing the vertex A.

4. The middle points of the sides of a triangle are $M_1(2,3)$, $M_2(-1,2)$ i $M_3(4,5)$. Find equations of the sides of the triangle.

5. Write an equation of the line such that the point P(2,3) is the projection of the origin on this line.

- 6. The lengths of vectors \vec{v} and \vec{w} are equal to 2 and 3, respectively. Knowing that $\vec{v} \circ \vec{w} = -1$
 - a) $(\vec{v} + 2\vec{w}) \circ (2\vec{v} \vec{w});$
 - b) cosine of the angle between $\vec{v} + \vec{w}$ and $\vec{v} \vec{w}$.

7. Check if the lines ℓ_1 and ℓ_2 are parallel. For parallel lines find the distance between them. For non-parallel lines find the acute angle between them.

a)
$$\ell_1 : x + y + 3 = 0, \ell_2 : \begin{cases} x = 1 - t, \\ y = 2 + t, \end{cases}$$
;
b) $\ell_1 : 2x - y + 1 = 0, \ell_2 : \begin{cases} x = 1 + t, \\ y = 2 - t, \end{cases}$