# ALGEBRA
## Homework List 2.
*Analytic geometry in the 3d space*

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**1.** Find the values of the parameters $t, s$ for which the vectors $\vec{v} = (2-2t, 2, -4)$ and $\vec{w} = (1, 3-s, 1)$ are parallel.

**2.** Find the values of the parameter $t$ for which vectors $\vec{v} = (2-2t, 2, -4)$ and $\vec{w} = (1, 3-t, 1)$ are perpendicular.

**3.** Compute the area of the parallelogram spanned by vectors $\vec{v} = (2, 2, -1)$ and $\vec{w} = (1, 3, 2)$.

**4.** Compute the area of the triangle with vertices $A = (1, 0, 1), B = (2, 0, 4)$ and $C = (0, 1, 1)$.

**5.** For the triangle from the previous problem calculate all altitudes.

**6.** Compute the volume of the parallelepiped spanned by vectors $\vec{u} = (2, 2, -4)$ $\vec{v} = (1, 2, 0)$ and $\vec{w} = (1, 3, 1)$.

**7.** Compute the volume of the tetrahedron with vertices $A = (0, 1, 0), B = (1, 1, 2), C = (0, 2, 1)$ and $D = (3, 2, -1)$.

**8.** For the tetrahedron from the previous problem compute the altitude through the vertex $D$.

**9.** Find normal and parametric equations of the plane

(a) through the points $P = (1, 2, 1), Q = (2, 1, 5)$ and $C = (3, 0, 1)$;

(b) through the point $P = (-2, 3, 2)$ and including the $Ox$ axis;

(c) through the point $P = (1, 0, 1)$ and perpendicular to the $Oy$ axis.

**10.** Do the parameteric equations

$$\begin{cases} x = 2 + 3t + s \\ y = 1 + t + 2s \\ z = -1 + t - s \end{cases} \quad \text{and} \quad \begin{cases} x = 5 + 4t + 2s \\ y = 2 + 3t + 4s \\ z = -2s \end{cases}$$

describe the same plane? Justify your answer.

**11.** Find a parametric equation of the plane given by the equation $x + 2y - z + 5 = 0$.

**12.** Find a normal equation of the plane given by the parametric equation

$$\begin{cases} x = 2 + t + 2s \\ y = 1 + 2t + s \\ z = 3 + t - s \end{cases}$$

**13.** Explain why the parametric equations

$$\begin{cases} x = 2 + t \\ y = 1 + t \\ z = -1 + 3t \end{cases} \quad \text{and} \quad \begin{cases} x = 2t \\ y = -1 + 2t \\ z = -7 + 6t \end{cases}$$

describe the same line.

**14.** Find a parametric equation of the line in which two planes

$$\begin{cases} x + 2y + z + 3 = 0 \\ 2x - y + z + 5 = 0 \end{cases}$$

intersect each other.

**15.** Find the intersection point of the line $l : x = t, y = 1 + 2t, z = 3 + t$ and the plane $\pi : x + 2y - z - 3 = 0$.

**16.** For the point $P = (1, 0, 1)$ and the plane $\pi : x + 2y - z + 3 = 0$, find

(a) the projection of $P$ on $\pi$;

(b) the distance from $P$ to $\pi$;

(c) the point, symmetric to $P$ with respect to $\pi$.

**17.** For the point $P = (1, 2, 3)$ and the line $l : x = 2t, y = 1 - t, z = -2 + 3t$, find

(a) the projection of $P$ on $l$;

(b) the distance from $P$ to $l$;

(c) the point, symmetric to $P$ with respect to $l$.

**18.** Find the distance between two parallel lines

$$\begin{cases} x + y + z + 2 = 0 \\ 2x - y + z + 5 = 0 \end{cases} \quad \text{and} \quad \begin{cases} x + y + z + 2 = 0 \\ 2x - y + z + 7 = 0 \end{cases}$$