

ALGEBRA
Homework list 3.
Complex numbers

1. Perform the algebraic operations and write the result in the algebraic form $x + iy$:

$$(a) (i + 3) - (2 - 3i); \quad (b) (1 - i)(2 + 5i); \quad (c) \frac{1 - 3i}{2 + 3i}; \quad (d) (1 - i)^4.$$

2. Comparing the real and imaginary parts of both sides of the equations, solve them for real x, y :

$$(a) (1 - i)x + (2 - i)y = 1 + i; \quad (b) \frac{x}{1 - i} + \frac{y}{1 + i} = 1 + i; \quad (c) 2x^2 + iy^2 = 3; \quad (d) 3x^2 - 2iy^2 = (1 + i)(i - 2).$$

3. Writing z in the algebraic form $z = x + iy$, solve the equations

$$(a) z^2 = -i; \quad (b) (3 - 2i)z = (2 + i); \quad (c) \frac{z + 1}{2 + i} = \frac{3 - z}{3 - 2i}; \quad (d) z^2 - 4z + 5;$$

$$(e) z(1 + i) + \bar{z}(2 - i) = 1 + i; \quad (f) i\operatorname{Re} z + \operatorname{Im} z = 1 + 2i; \quad (g) z\bar{z} = (\bar{z})^2.$$

4. Find all complex numbers z which satisfy the following conditions:

$$(a) \operatorname{Re} z + \operatorname{Im} z = 3; \quad (b) \operatorname{Re}(-iz) \leq 1; \quad (c) \operatorname{Im}((1 + i)z) \leq 2.$$

Indicate the solution on the complex plane.

5. Write the following numbers in the trigonometric form:

$$(a) 2i; \quad (b) -1 + \sqrt{3}i; \quad (c) -2\sqrt{3} - 2i; \quad (d) \left(\frac{1 - \sqrt{3}i}{2 + 2\sqrt{3}i} \right)^5.$$

6. Using de Moivre's formula, compute the following powers:

$$(a) (1 - i)^{13}; \quad (b) (-1 + \sqrt{3}i)^{15}; \quad \left(\frac{1 + i}{-1 + i\sqrt{3}} \right)^{17}.$$

Give the answers in the algebraic form.

7. Using the trigonometric form of complex numbers, compute the following roots:

$$(a) \sqrt[6]{-1}; \quad (b) \sqrt[3]{-\sqrt{3} + i}; \quad (c) \sqrt[6]{-64}.$$

Give the answers in the algebraic form.

8. Draw on the complex plane the sets of complex numbers satisfying the following conditions:

$$(a) |2z + i| = 6; \quad (b) |3z - 1| < 3; \quad (c) 2 \leq |2z + i| \leq 4; \quad (d) |z - 2i| = |z + i|;$$

$$(e) \operatorname{Im}(z^3) < 0; \quad (f) \operatorname{Re}(z^4) \geq 0.$$

9. Using the algebraic form of complex numbers, compute the following roots:

$$(a) \sqrt{1 - 2i}; \quad (b) \sqrt{5 - i}.$$

10. Solve the equations for complex z :

$$(a) z^2 - z + 1 = 0; \quad (b) z^2 + 16 = 0; \quad (c) z^4 - 3z^2 + 2 = 0; \quad (d) z^2 + (1 - i)z + 2i = 0; \quad (e) z^4 = -1;$$

$$(f) z^2 + 4iz + 1 = 0; \quad (g) z^3 = (1 + i)^3; \quad (h) (z - i)^4 = (2z + 1)^4.$$