ALGEBRA
Homework List 4.
Matrices and linear mappings

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1. Let A, B, C be matrices defined by

$$A = \begin{pmatrix} 3 & 0 & 1 \\ -1 & 2 & 0 \\ 1 & 1 & -1 \end{pmatrix}, \quad B = \begin{pmatrix} -1 & 2 & 1 \\ 1 & 0 & 1 \\ 4 & 3 & -1 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 2 \\ 2 & 1 \\ -1 & 1 \end{pmatrix}.$$

Which of the matrices: $A+B, A+C, 2A, AB, BA, AC, CA, A^2, C^2$ are well defined? Compute the matrices which are well defined.

2. Let A, B be matrices defined by

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 0 \\ -1 & 2 \end{pmatrix}.$$

Compute and compare AB and BA.

3. The linear mapping of $\mathbb{R}^2$ transforms the vector $(1, 2)$ to $(-1, 1)$, and the vector $(2, 1)$ to $(3, 1)$. Write the matrix of this mapping in the standard basis in $\mathbb{R}^2$.

4. . Let the linear mapping of $\mathbb{R}^2$ be given by $T(x, y) = (2x + y, x - y)$. Find its matrices in the standard basis $B = \{e_1, e_2\}$ and in the basis $B' = \{v_1, v_2\}$ given by $v_1 = (1, 1), v_2 = (1, -1)$.

5. Define the linear mapping of $\mathbb{R}^2$ which corresponds to rotation clockwise around the origin by the angle α composed with the reflection with respect to Ox axis. Write the matrix of this mapping in the standard basis in $\mathbb{R}^2$.

6. Define the linear mapping of $\mathbb{R}^2$ which corresponds to reflection with respect to

(a) the Oy axis;

(b) the line $y + x = 0$;

(c) the line $3y - 4x = 0$.

Write the matrices of these mappings in the standard basis in $\mathbb{R}^2$.

7. Define the linear mapping of $\mathbb{R}^3$ which corresponds to reflection with respect to

(a) the Oz axis;

(b) the Oyz plane;

(c) the plane $x + 2y - 3z = 0$.

Write the matrices of these mappings in the standard basis in $\mathbb{R}^3$.

8. Define the linear mappings of $\mathbb{R}^3$ which corresponds to rotation counter-clockwise around the Oy and Oz axes by the angle α. Write the matrices of these mappings in the standard basis in $\mathbb{R}^3$. For which values of α these mappings commute?