ALGEBRA

Homework List 5.

Determinants, systems of linear equations

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1. Write the Laplace expansions of the given determinants along indicated rows or columns

$$\begin{pmatrix} 2 & 1 & \mathbf{2} \\ 3 & 2 & \mathbf{1} \\ 4 & 3 & \mathbf{-1} \end{pmatrix}, \quad \begin{pmatrix} 2 & 1 & 3 & 1 \\ \mathbf{1} & \mathbf{0} & \mathbf{2} & \mathbf{1} \\ 1 & -1 & 1 & 3 \\ 2 & -2 & 1 & -3 \end{pmatrix}, \quad \begin{pmatrix} 1 & -3 & 1 & \mathbf{2} \\ 2 & 3 & -2 & \mathbf{1} \\ -2 & 1 & 1 & \mathbf{0} \\ 1 & 4 & 3 & \mathbf{0} \end{pmatrix}$$

2. Calculate the determinants from the previous problem.

3. Calculate the determinants

$$\begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix}, \quad \begin{pmatrix} 1 & 2 & -3 \\ -2 & 3 & -1 \\ 3 & 2 & 1 \end{pmatrix}, \quad \begin{pmatrix} 2 & 0 & 2 & 3 \\ -1 & 1 & -1 & 1 \\ -2 & 0 & 2 & 0 \\ 5 & -1 & -1 & 1 \end{pmatrix}.$$

4. Using the properties of the determinants, justify that the following matrices are not invertible

$$\begin{pmatrix} 1 & -2 & 3 \\ 0 & 1 & -1 \\ -1 & 0 & -1 \end{pmatrix}, \quad \begin{pmatrix} 1 & 1 & -1 & -1 \\ 3 & 2 & 1 & 3 \\ 4 & 5 & 6 & 7 \\ 0 & 2 & 6 & 5 \end{pmatrix}.$$

5. Calculate the determinants in Problem 3, using the Gauss elimination algorithm.

6. Using the cofactor formula, compute the inverses of the following matrices:

$$\begin{pmatrix} 2 & 3 \\ 4 & 1 \end{pmatrix}, \quad \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 1 & -1 \end{pmatrix}, \quad \begin{pmatrix} 0 & 0 & 1 & 0 \\ 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 4 & 0 & 0 \end{pmatrix}.$$

7. Using inverse matrices, solve the following matrix equations:

$$\begin{pmatrix} 2 & 3 \\ 4 & 1 \end{pmatrix} \cdot X = \begin{pmatrix} 1 & 1 & -1 \\ 2 & -3 & 1 \end{pmatrix}, \quad \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 1 & -1 \end{pmatrix} \cdot X = \begin{pmatrix} 1 & -1 \\ 2 & 3 \\ 0 & 2 \end{pmatrix}.$$

8. Applying Cramer's Rule to the following systems of equations, compute the indicated unknown:

$$(a) \quad \begin{cases} x + y = 1 \\ 2x - 3y = 2 \end{cases}, \text{ unknown } x \qquad (b) \quad \begin{cases} x + 2y + z = 1 \\ x - 2y - z = -2 \\ 2x + y + 4z = -1 \end{cases}, \text{ unknown } z.$$

9. Applying the Gauss elimination method, solve the following systems of equations

$$\begin{cases} x + 2y + z = 3 \\ 3x + 2y + z = 3 \\ x - 2y - 5z = 1 \end{cases}$$

$$\begin{cases} x + 2y + 3z - 4v = 0 \\ 2x - y + 3z - 2v = 2 \\ 3x + 4z + 2v = -1 \end{cases}$$

10. Calculate the inverses of the matrices from Problem 6 using the Gauss elimination method.

11. Solve the matrix equations from Problem 7 using the Gauss elimination method.

12. Applying the Kronecker-Capelli theorem, show that the system

$$\begin{cases} x + y + 2z - v = -1 \\ 2x + y + 3z + v = 3 \\ 3x + y - z - 2v = -4 \end{cases}$$

has infinitely many solutions and then solve this system.

13. Applying the Kronecker-Capelli theorem, show that the system

$$\begin{cases} x + 2y + z + 4v = 1 \\ 2x + y + 3v = 3 \\ -x + 2z + v = 1 \\ 2x + y + 3z + 6v = 6 \end{cases}$$

is inconsistent.