ALGEBRA
Homework List 6.
Eigenvalues and eigenvectors

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1. Determine the real eigenvalues and eigenvectors of the following matrices:

$$\begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix}, \quad \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}, \quad \begin{pmatrix} -1 & -1 & -2 \\ 0 & 2 & 2 \\ 0 & -1 & -1 \end{pmatrix}, \quad \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}.$$

2. Find the complex eigenvalues and eigenvectors of the following matrices:

$$\begin{pmatrix} 1 & 1 \\ -2 & 3 \end{pmatrix}, \quad \begin{pmatrix} 1 & -4 \\ 1 & 1 \end{pmatrix}, \quad \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -1 & 1 \end{pmatrix}, \quad \begin{pmatrix} 1 & 2 & 0 \\ -1 & 3 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}.$$

3. Find the eigenvalues and eigenvectors of the following linear mappings:

(a) $T : \mathbb{R}^2 \to \mathbb{R}^2$, where $T(x, y) = (x - 2y, x + y)$;

(b) $T : \mathbb{R}^3 \to \mathbb{R}^3$, where $T(x, y, z) = (2z, x, y)$;

(c) $T : \mathbb{C}^3 \to \mathbb{C}^3$, where $T(x, y, z) = (x + y + z, y + z, z)$.

4. Let T be the orthogonal projection in the space $\mathbb{R}^2$ on the x axis. Using the geometric interpretation of T, determine its eigenvalues and eigenvectors.

5. Diagonalize the real matrices

$$\begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}, \quad \begin{pmatrix} -1 & 0 & -4 \\ 0 & -1 & 0 \\ 2 & -4 & 5 \end{pmatrix}, \quad \begin{pmatrix} 2 & -1 & -1 \\ 3 & -2 & -3 \\ -1 & 1 & 2 \end{pmatrix}, \quad \begin{pmatrix} -5 & 0 & -2 \\ 4 & -1 & 2 \\ 4 & 0 & 1 \end{pmatrix},$$