

ALGEBRA
List 3.
Complex numbers

1. Perform the algebraic operations and write the result in the algebraic form $x + iy$:

$$(a) (5 + i) - (3 - 2i); \quad (b) (1 + i)(2 - 3i); \quad (c) \frac{2 - i}{3 + 2i}; \quad (d) (1 + i)^4.$$

2. Comparing the real and imaginary parts of both sides of the equations, solve them for real x, y :

$$(a) (1 + i)x + (1 - 2i)y = 1 - i; \quad (b) \frac{x}{1 + i} + \frac{y}{1 - i} = 2 + i; \quad (c) x^2 + 2iy^2 = i; \quad (d) x^2 - iy^2 = 1 + i.$$

3. Writing z in the algebraic form $z = x + iy$, solve the equations

$$(a) z^2 = i; \quad (b) (3 + 2i)z = (1 + i); \quad (c) \frac{z + 2}{2 - i} = \frac{1 - z}{3 + 2i}; \quad (d) z^2 - 6z + 10;$$

$$(e) z(1 - i) + \bar{z}(2 + i) = 3 - i; \quad (f) z\bar{z} + (z - \bar{z}) = 3 + 2i; \quad (g) i\operatorname{Re} z + \operatorname{Im} z = 2 + i; \quad (g) z\bar{z} = z^2.$$

4. Find all complex numbers z which satisfy the following conditions:

$$(a) \operatorname{Re} z - 3\operatorname{Im} z = 2; \quad (b) \operatorname{Re}(iz) \geq 1; \quad (c) \operatorname{Im}(iz) \leq 2.$$

Indicate the solution on the complex plane.

5. Write the following numbers in the trigonometric form:

$$(a) -3i; \quad (b) 1 + \sqrt{3}i; \quad (c) 2 - 2\sqrt{3}i; \quad (d) \left(\frac{1 + \sqrt{3}i}{2 - 2\sqrt{3}i} \right)^3.$$

6. Using de Moivre's formula, compute the following powers:

$$(a) (1 + i)^{11}; \quad (b) (2 - 2\sqrt{3}i)^{10}; \quad \left(\frac{1 - i}{1 - i\sqrt{3}} \right)^{12}.$$

Give the answers in the algebraic form.

7. Using the trigonometric form of complex numbers, compute the following roots:

$$(a) \sqrt[6]{1}; \quad (b) \sqrt[3]{\sqrt{3} + i}; \quad (c) \sqrt[4]{-16}.$$

Give the answers in the algebraic form.

8. Draw on the complex plane the sets of complex numbers satisfying the following conditions:

$$(a) |z + i| = 5; \quad (b) |z - 1| < 3; \quad (c) 1 \leq |z + i| \leq 2; \quad (d) |z - i| = |z + i|;$$

$$(e) \operatorname{Re}(z^3) < 0; \quad (f) \operatorname{Im}(z^4) \geq 0.$$

9. Using the algebraic form of complex numbers, compute the following roots:

$$(a) \sqrt{2 - i}; \quad (b) \sqrt{3 - 2i}.$$

10. Solve the equations for complex z :

$$(a) z^2 + z + 1 = 0; \quad (b) z^2 + 9 = 0; \quad (c) z^4 - 2z^2 + 4 = 0; \quad (d) z^2 + (1 + i)z - i = 0; \quad (e) z^4 = 1;$$

$$(f) z^2 + 3iz + 4 = 0; \quad (g) z^3 = (1 - i)^3; \quad (h) (z - i)^4 = (iz + 4)^4.$$