ALGEBRA
List 4.
Matrices and linear mappings

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1. Let A, B, C be matrices defined by

$$A = \begin{pmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 5 & 2 \\ -1 & 1 & 0 \\ -4 & 3 & 1 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 1 \\ 2 & 2 \\ 0 & 1 \end{pmatrix}.$$

Which of the matrices: $A+B, A+C, 2A, AB, BA, AC, CA, A^2, B^2$ are well defined? Compute the matrices which are well defined.

2. Let A, B be matrices defined by

$$A = \begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 0 \\ 2 & 3 \end{pmatrix}.$$

Compute and compare AB and BA.

3. The linear mapping of $\mathbb{R}^2$ transforms the vector $(1,1)$ to $(-1,2)$, and the vector $(1,0)$ to $(1,3)$. Write the matrix of this mapping in the standard basis in $\mathbb{R}^2$.

4. . Let the linear mapping of $\mathbb{R}^2$ be given by $T(x,y) = (x+y, 3x-y)$. Find its matrices in the standard basis $B = \{e_1, e_2\}$ and in the basis $B' = \{v_1, v_2\}$ given by $v_1 = (1,1), v_2 = (1,-1)$.

5. Define the linear mapping of $\mathbb{R}^2$ which corresponds to rotation counter-clockwise around the origin by the angle α. Write the matrix of this mapping in the standard basis in $\mathbb{R}^2$.

6. Define the linear mapping of $\mathbb{R}^2$ which corresponds to reflection with respect to

(a) the Ox axis;

(b) the line $y = x$;

(c) the line $y + 2x = 0$.

Write the matrices of these mappings in the standard basis in $\mathbb{R}^2$.

7. Define the linear mapping of $\mathbb{R}^3$ which corresponds to reflection with respect to

(a) the Ox axis;

(b) the Oxy plane;

(c) the plane $y + 2x + z = 0$.

Write the matrices of these mappings in the standard basis in $\mathbb{R}^3$.

8. Define the linear mapping of $\mathbb{R}^3$ which corresponds to rotation counter-clockwise around the Ox axis by the angle α. Write the matrix of this mapping in the standard basis in $\mathbb{R}^3$.

9. Let T be the linear mapping of $\mathbb{R}^3$ which corresponds to rotation counter-clockwise around the Oy axis by the angle $\frac{\pi}{3}$, and U be the linear mapping of $\mathbb{R}^3$ which corresponds to rotation counter-clockwise around the Oz axis by the angle $\frac{\pi}{6}$. Write the matrices of these mappings in the standard basis in $\mathbb{R}^3$. Verify, whether the composition mappings $T \circ U$ and $U \circ T$ are equal or not.