ALGEBRA

List 5.

Determinants, systems of linear equations

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1. Write the Laplace expansions of the given determinants along indicated rows or columns

$$\begin{pmatrix} -1 & 4 & \mathbf{3} \\ -3 & 1 & \mathbf{0} \\ 2 & 5 & \mathbf{-2} \end{pmatrix}, \quad \begin{pmatrix} 1 & 2 & 3 & 0 \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{2} \\ 2 & 3 & 3 & 0 \\ 1 & 2 & 3 & 1 \end{pmatrix}, \quad \begin{pmatrix} 1 & 3 & 2 & \mathbf{1} \\ 2 & 4 & -1 & \mathbf{0} \\ -1 & 0 & 2 & \mathbf{0} \\ 3 & 2 & 5 & \mathbf{-1} \end{pmatrix}$$

2. Calculate the determinants from the previous problem.

3. Calculate the determinants

$$\begin{pmatrix} -2 & 4 \\ -3 & 1 \end{pmatrix}, \quad \begin{pmatrix} 1 & 2 & 3 \\ -1 & 1 & -1 \\ 2 & 1 & 3 \end{pmatrix}, \quad \begin{pmatrix} 1 & 0 & 2 & 1 \\ 2 & 1 & -1 & 3 \\ -1 & 0 & 2 & 0 \\ 3 & 2 & 1 & 1 \end{pmatrix}.$$

4. Using the properties of the determinants, justify that the following matrices are not invertible

$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & -1 \\ -2 & -4 & -6 \end{pmatrix}, \quad \begin{pmatrix} 1 & 3 & 2 & 1 \\ 4 & 2 & 1 & 3 \\ 3 & 3 & 1 & 2 \\ 0 & 4 & 2 & 0 \end{pmatrix}.$$

5. Calculate the determinants in Problem 3, using the Gauss elimination algorithm.

6. Using the cofactor formula, compute the inverses of the following matrices:

$$\begin{pmatrix} -2 & 4 \\ -3 & 1 \end{pmatrix}, \quad \begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & 2 & -1 \end{pmatrix}, \quad \begin{pmatrix} 0 & 1 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 4 & 0 \end{pmatrix}.$$

7. Using inverse matrices, solve the following matrix equations:

$$\begin{pmatrix} 3 & 5 \\ 1 & 2 \end{pmatrix} \cdot X = \begin{pmatrix} 0 & 1 & 1 \\ 2 & 3 & -1 \end{pmatrix}, \quad \begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & 2 & -1 \end{pmatrix} \cdot X = \begin{pmatrix} 1 & 2 \\ 3 & 1 \\ 2 & 0 \end{pmatrix}.$$

8. Applying Cramer's Rule to the following systems of equations, compute the indicated unknown:

$$(a) \quad \begin{cases} 2x - y = 0 \\ 3x + 2y = 5 \end{cases}, \text{ unknown } y \quad (b) \quad \begin{cases} x + y + 2z = -1 \\ 2x - y + 2z = -4 \\ 4x + y + 4z = -2 \end{cases}, \text{ unknown } x.$$

9. Applying the Gauss elimination method, solve the following systems of equations

$$\begin{cases} x + 2y + z = 3 \\ 3x + 2y + z = 3 \\ x - 2y - 5z = 1 \end{cases}$$

$$\begin{cases} x + 2y + 4z - 3v = 0 \\ 3x + 5y + 6z - 4v = 1 \\ 4x + 5y - 2z + 3v = 1 \end{cases}$$

10. Calculate the inverses of the matrices from Problem 6 using the Gauss elimination method.

11. Solve the matrix equations from Problem 7 using the Gauss elimination method.

12. Applying the Kronecker-Capelli theorem, show that the system

$$\begin{cases} x + 2y + 3z - v = -1 \\ 3x + 6y + 7z + v = 5 \\ 2x + 4y + 7z - 4v = -6 \end{cases}$$

has infinitely many solutions and then solve this system.

13. Applying the Kronecker-Capelli theorem, show that the system

$$\begin{cases} x - y - 2z + 2v = -2 \\ 5x - 3y - z + v = 3 \\ 2x + y - z + v = 1 \\ 3x - 2y + 2z - 2v = -4 \end{cases}$$

is inconsistent.