

ALGEBRA
List 6.
Eigenvalues and eigenvectors

1. Determine the real eigenvalues and eigenvectors of the following matrices:

$$\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}, \quad \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}, \quad \begin{pmatrix} 1 & 1 & 0 \\ 2 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \begin{pmatrix} 0 & 1 & 0 \\ -2 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}.$$

2. Find the complex eigenvalues and eigenvectors of the following matrices:

$$\begin{pmatrix} 1 & -5 \\ 2 & -1 \end{pmatrix}, \quad \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}, \quad \begin{pmatrix} 0 & 8 & 0 \\ 0 & 0 & -2 \\ 2 & 8 & -2 \end{pmatrix}, \quad \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}, \quad \begin{pmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}.$$

3. Find the eigenvalues and eigenvectors of the following linear mappings:

(a) $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$, where $T(x, y) = (x + 2y, x - y)$;

(b) $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$, where $T(x, y, z) = (y, x, z)$;

(c) $T : \mathbb{C}^3 \rightarrow \mathbb{C}^3$, where $T(x, y, z) = (x + 2y + z, -2x + y, z)$.

4. Let T be the reflection of the space \mathbb{R}^2 with respect to the x axis. Using the geometric interpretation of T , determine its eigenvalues and eigenvectors.

5. Diagonalize the real matrices

$$\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}, \quad \begin{pmatrix} 1 & 1 & 0 \\ 2 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad \begin{pmatrix} 4 & 0 & 6 \\ 2 & 1 & 4 \\ -1 & 0 & -1 \end{pmatrix},$$