

ALGEBRA

List 7.

Polynomials, rational functions, partial fractions

1. Find all integer roots of the following real polynomials:

$$(a) x^3 + 3x^2 - 4; \quad (b) x^4 - 2x^3 - 8x - 12; \quad (c) x^4 - x^2 - 2.$$

2. Find all rational roots of the following real polynomials:

$$(a) 4x^4 + 4x^3 + 3x^2 - x - 1; \quad (b) x^4 - 5x^2 + 4; \quad (c) 6x^4 + 7x^2 + 2.$$

3. Find all roots of the following real polynomials:

$$(a) x^4 + x^3 - 3x^2 - 4x - 4; \quad (b) x^4 + x^3 + x^2 + 3x - 6.$$

4. Find all roots of the following complex polynomials, knowing one of their roots:

$$(a) z^4 - 4z^3 + 12z^2 - 16z + 15, z_1 = 1 + 2i; \quad (b) z^4 - 4z^3 + 16z^2 - 12z + 39, z_1 = i\sqrt{3}.$$

5. Perform the long division and find $D(x), R(x)$ such that $P(x) = D(x)Q(x) + R(x)$, $\deg(R) < \deg(Q)$ for

$$(a) P(x) = x^{10} - 2x^7 + 3x, Q(x) = x^2 + 1; \quad (b) P(x) = 2x^9 - 3x^5 + 5, Q(x) = x^3 + x^2 - 1.$$

6. Factor the following real polynomials into irreducible real factors:

$$(a) x^3 + x^2 + x + 1; \quad (b) x^4 - 4x^3 + 8x; \quad (c) x^4 + 5x^2 + 6.$$

7. Factor the following complex polynomials into irreducible real factors:

$$(a) z^3 + z^2 + z + 1; \quad (b) z^4 + 5z^2 + 6; \quad (c) z^6 + 8.$$

8. Decompose the following real rational functions into real partial fractions:

$$(a) \frac{x}{(x^2 + 1)(x - 2)}; \quad (b) \frac{x - 1}{x^3 - 1}; \quad (c) \frac{1}{(x^2 + 1)(x - 1)(x + 2)}.$$

9. Decompose the following complex rational functions into complex partial fractions:

$$(a) \frac{1}{z^3 - 2z^2 + z - 2}; \quad (b) \frac{z^2}{z^3 - 1}; \quad (c) \frac{1}{(z^2 + 1)(z - 1)}.$$