LISTS OF QUESTIONS FOR THE MIDTERM TEST FOR THE COURSE MATH-ALGEBRA

1. Complex numbers. Basic operations, modulus, complex conjugate.

2. Polar (trigonometric) form of complex number. Multiplication and division in polar form. De Moivre's formula.

3. Powers and roots of complex numbers.

4. The notion of algebraic field. Examples.

5. Polynomials. Roots of polynomials, the Bésout (polynomial remainder) theorem.

6. Fundamental theorem of algebra. Decomposition of complex and real polynomials into irreducible factors.

7. Decomposition of complex and real rational functions into partial fractions.

8. Vectors in $\mathbf{R}^2$ and $\mathbf{R}^3$: definition, multiplication by scalars, addition, modulus.

9. Scalar product of vectors in $\mathbf{R}^2$ and $\mathbf{R}^3$: definition, bilinearity, Cauchy-Schwartz inequality. Angle between two vectors.

10. Vector and mixed products in $\mathbf{R}^3$: definition, properties, associated formulae for areas and volumes.

11. Equation of a line in $\mathbf{R}^2$: parametric (directional), general (normal), and slope-intercept forms. Parametric equation of a line in $\mathbf{R}^3$

12. Equation of a plane in $\mathbf{R}^3$: parametric and general (normal) forms, equation in segments. Normal vector to a plane.

13. Vector space: definition, axioms, examples.

14. Linear combinations, spanning, and linear independence. Basis of a vector space. Dimension of a vector space.

15. Matrices: definition, addition and multiplication. Properties of the matrix multiplication.

16. Permutations and determinants: definitions. Minors and cofactors.

17. Laplace's formula for determinant.

18. The rank of the matrix.

19. The echelon form and the reduced echelon form of a matrix.

20. Systems of linear equations: matrix notation, elementary operations, the Gauss algorithm.

21. Homogeneous and non-homogeneous linear systems. The Kronecker-Capelli theorem.

22. Cramer's formulas.