MATHEMATICAL ANALYSIS I

Exam, version 3.

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1. (2+5p.) Theorem about the derivative of a composition of functions (the chain rule): formulation, examples. Find the minimal and maximal values of the function $f(x) = \dfrac{x-1}{x^2+1}$ on the interval $[1,3]$.

2. (3+4p.) Monotonicity and convexity: definitions, necessary and sufficient conditions in the terms of derivatives. Write the equation for the tangent line to the graph of the function $f(x) = \log_3\left(x^2+1\right)$ at the point $x = -2$.

3. (2+5p.) The change of variables formula for definite integral. Calculate the indefinite integral $\displaystyle\int \frac{x^2}{2^x}\,dx$.

4. (2+5p.) Write the formulae for the volume of the bodies of revolution of the curve $y = f(x)$ w.r.t. the axis Ox and Oy. Calculate the limit $\displaystyle\lim_{x\to+\infty} \left(\frac{2\arctan x}{\pi}\right)^{\sin\frac{1}{x}}$

5. (2+5p.) Write the Taylor formula of the order n with the residual term in the Lagrange form. Find the integral $\displaystyle\int \frac{x^2+2x}{x^4+x^2-2}\,dx$