

# MATHEMATICAL ANALYSIS I

Exam, version 7.

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- 1. (2+5p.)** Theorem about the derivative of a composition of functions (the chain rule): formulation, examples. Find the minimal and maximal values of the function  $f(x) = \frac{x-1}{x^2+1}$  on the interval  $[1, 3]$ .
- 2. (3+4p.)** Monotonicity and convexity: definitions, necessary and sufficient conditions in the terms of derivatives. Write the equation for the tangent line to the graph of the function  $f(x) = \log_3(x^2 + 1)$  at the point  $x = -2$ .
- 3. (2+5p.)** The change of variables formula for definite integral. Calculate the indefinite integral  $\int \frac{x^2}{2^x} dx$ .
- 4. (2+5p.)** Write the formulae for the volume of the bodies of revolution of the curve  $y = f(x)$  w.r.t. the axis  $Ox$  and  $Oy$ . Calculate the limit  $\lim_{x \rightarrow +\infty} \left( \frac{2 \arctan x}{\pi} \right)^{\sin \frac{1}{x}}$
- 5. (2+5p.)** Write the Taylor formula of the order  $n$  with the residual term in the Lagrange form. Find the integral  $\int \frac{x^2 + 2x}{x^4 + x^2 - 2} dx$