MATHEMATICAL ANALYSIS 2

Exam retake, version 1.

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1. (2+5p.) Write the definition of the gradient of a function of two variables. Write the definition of the directional derivative. How these two objects are related? Find and classify all the critical points of $f(x,y) = x^3 + x^2y + 2x^2 + y^2$.

2. (3+3p.) Write the definitions of a global extremum, local extremum, and local extremum. Give an example of a local extremum which is not a global one. Find all the vectors $\mathbf{v} \in \mathbb{R}^3$ such that the directional derivative of the function $f(x,y,z) = \dfrac{\ln(2+x)}{\sqrt{y^3+z^2}}$ at the point $(0,1,-1)$ in the direction $\mathbf{v}$ equals 0.

3. (3+4p.) Write the definition a normal domain on the plane. Draw an example of y-normal domain which is not x-normal. Calculate the double integral $\iint_D \frac{y}{x} dxdy$, where the domain D is bounded by the curves $y = x^2, y = \sqrt[3]{x}$.

4. (2+5p.) Write the change of variables formula in a double integral. Performing a proper change of variables, calculate

$$\iint_D (x-y)^2 \; dxdxy, \quad D = \{(x,y) : x^2 + y^2 \leqslant 4, x \leqslant y \leqslant -\sqrt{3}x\}.$$

Draw the domain of integration in (x,y)- and new coordinates.

5. (2+5p.) Write the Taylor formula with the residue term in the Lagrange form. Write the Taylor series for the function $f(x) = (2+x)^{-2}$ at the point $x_0 = 2$. Find the radius of convergence and the interval of convergence of this series.