MATHEMATICAL ANALYSIS 2

Exam retake, version 2.

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1. (2+5p.) Write the definitions of a partial derivative and a directional derivative. Find and classify all the critical points of $f(x, y) = x^3 - x^2y - 2x^2 - y^2$.

2. (3+3p.) Explain the Lagrange multipliers method. Find all the vectors $\mathbf{v} \in \mathbb{R}^3$ such that the directional derivative of the function $f(x, y, z) = \dfrac{\cos x^2}{\sqrt{y + z^2}}$ at the point $(0, -1, 2)$ in the direction $\mathbf{v}$ equals 0.

3. (3+4p.) Write the definition a normal domain on the plane. Draw an example of y-normal domain which is not x-normal. Calculate the double integral $\iint_D xydxdy$, where the domain D is bounded by the curves $y = \sqrt{x}, y = x^2$.

4. (2+5p.) Write the definitions of the Jacobian matrix and the Jacobian determinant. Performing a proper change of variables, calculate

$$\iint_D (x + y)^2 \, dxdxy, \quad D = \{(x, y) : x^2 + y^2 \leqslant 2, -x \leqslant y \leqslant -\sqrt{3}x\}.$$

Draw the domain of integration in (x, y)- and new coordinates.

5. (2+5p.) Write the Taylor formula with the residue term in the Lagrange form. Write the Taylor series for the function $f(x) = (2x - 3)^{-2}$ at the point $x_0 = 1$. Find the radius of convergence and the interval of convergence of this series.