

## MATHEMATICAL ANALYSIS 2

### Exam retake, version 3.

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1. (2+5p.) Write the equation for the tangent plane for a function  $f(x, y)$ . Find and classify all the critical points of  $f(x, y) = x^3 + xy + x^2 + y^2$ .

2. (3+3p.) Write the definition of positive defined and negative defined matrices. Formulate the Sylvester criterion. Find the directional derivative of the function  $f(x, y, z) = \frac{\cos x^2}{\sqrt{y + z^2}}$  at the point  $(0, 2, -1)$  in the direction  $(1, 1, 2)$ .

3. (3+4p.) List the properties of a double integral. Calculate the double integral  $\iint_D \frac{y}{x} dx dy$ , where the domain  $D$  is bounded by the curves  $y = -\sqrt{x}$ ,  $y = x^2$ ,  $x = 3$ .

4. (2+5p.) Write the change of variables formula for polar coordinates. Performing a proper change of variables, calculate

$$\iint_D (x + y)^2 dx dy, \quad D = \{(x, y) : x^2 + y^2 \leq 3, -x \leq y \leq \sqrt{3}x\}.$$

Draw the domain of integration in  $(x, y)$ - and new coordinates.

5. (2+5p.) Write the formula for the Taylor-Maclaurin series. Write the Taylor series for the function  $f(x) = (x - 2)^{-2}$  at the point  $x_0 = 5$ . Find the radius of convergence and the interval of convergence of this series.