MATHEMATICAL ANALYSIS 2

Exam retake, version 4.

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1. (2+5p.) Write the equation for the tangent plane for a function $f(x,y)$. Find and classify all the critical points of $f(x,y) = y^3 - xy - x^2 - y^2$.

2. (3+3p.) Write the definitions of a global extremum, local extremum, and local extremum. Give an example of a local extremum which is not a global one. Find all the vectors $\mathbf{v} \in \mathbb{R}^3$ such that the directional derivative of the function $f(x,y,z) = \dfrac{\sin x}{\sqrt{y^2 + z^3}}$ at the point $(0,-1,1)$ in the direction $\mathbf{v}$ equals 0.

3. (3+4p.) Write the definition of a normal and regular domains on the plane. Draw an example of x-normal domain which is not y-normal. Calculate the double integral $\iint_D \frac{y}{x} dxdy$, where the domain D is bounded by the curves $y = -\sqrt{x}, y = \sqrt[5]{x}, x = 1$.

4. (2+5p.) Write the change of variables formula in double integral. Performing a proper change of variables, calculate

$$\iint_D (x - 2y)^2 \; dxdxy, \quad D = \{(x,y) : x^2 + y^2 \leqslant 1, -x \leqslant y, x \leqslant -\sqrt{3}y\}.$$

Draw the domain of integration in (x,y)- and new coordinates.

5. (2+5p.) Write the Taylor formula of the order n. Write the Taylor series for the function $f(x) = (2+x)^{-2}$ at the point $x_0 = 4$. Find the radius of convergence and the interval of convergence of this series.