LISTS OF QUESTIONS FOR THE FINAL EXAM FOR THE COURSE ELEMENTARY LINEAR ALGEBRA 1

1. Complex numbers. Basic operations, modulus, complex conjugate.
2. Polar (trigonometric) form of complex number. Multiplication and division in polar form. De Moivre's formula.
3. Powers and roots of complex numbers.
4. The notion of algebraic field. Examples.
5. Polynomials. Addition and multiplication of polynomials. Long division algorithm.
6. Roots of polynomials, the Bésout (polynomial remainder) theorem. The rational roots theorem.
7. Fundamental theorem of algebra. Decomposition of complex and real polynomials into irreducible factors.
8. Decomposition of complex and real rational functions into irreducible factors.
9. Vectors in $\mathbf{R}^2$ and $\mathbf{R}^3$: definition, multiplication by scalars, addition, modulus.
10. Scalar product of vectors in $\mathbf{R}^2$ and $\mathbf{R}^3$: definition, bilinearity, Cauchy-Schwartz inequality. Angle between two vectors.
11. Vector and mixed products in $\mathbf{R}^3$: definition, properties, associated formulae for areas and volumes.
12. Equation of a line in $\mathbf{R}^2$: parametric (directional), general (normal), and slope-intercept forms. Parametric equation of a line in $\mathbf{R}^3$
13. Equation of a plane in $\mathbf{R}^3$: parametric and general (normal) forms, equation in segments. Normal vector to a plane.
14. Vector space: definition, axioms, examples.
15. Linear combinations, spanning, and linear independence. Basis of a vector space. Dimension of a vector space.
16. Linear mappings between vector spaces. Matrix notation for linear mappings. Change of the matrix of a transformation under the change of a basis.
17. Matrices: definition, addition and multiplication. Examples of matrices.
18. Systems of linear equations: matrix notation, elementary operations, the Gauss algorithm.
19. The rank of the matrix.
20. Homogeneous and non-homogeneous linear systems. The Capelli theorem.
21. Permutations and determinants: definitions. Minors and cofactors.
22. Laplace's formula for determinant.
23. Cramer's formulas.
24. The kernel and the range of a linear transformation: definition, relation to the rank of the matrix of the transformation.
25. Eigenvalues and eigenvectors. Eigenbasis and diagonalization of a matrix.

26. Euclidean spaces. The Gram matrix of a scalar product in a given basis.
27. Orthonormal bases. The Gram-Schmidt orthogonalization procedure.
28. Orthogonal matrices: definition, properties. Diagonalization of a symmetric matrix.
29. Orthogonal complement of a linear subspace: definition, properties.