Mathematical Analysis II
Question List for the final Exam.

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1. Partial derivatives of a function of several variables: definition, examples.
2. Directional derivatives: definition, relation to partial derivatives.
3. Gradient, its relation to partial and directional derivatives.
4. The formula of small increments
5. The tangent plane to a graph of a function: definition, equation.
6. Higher order partial derivatives: definition, examples.
7. The Schwartz lemma.
8. The Hessian of a function: definition, relation to second derivative of a section of a function in a given direction.
9. Sufficient condition for local convexity/concavity.
10. Positive/negative definiteness of a symmetric matrix: definition and the Sylvester criterium.
11. Local extrema: definition and necessary condition in the terms of the gradient
12. Classification of critical points. Sufficient conditions for a critical point to be a local minimum/local maximum
13. Local extrema under given constraints: definition, the Lagrange multipliers method.
14. The integral of a function of two variables over a rectangle: definition, properties.
15. Normal and regular domains: definitions, examples.
16. Representation of an integral over a normal domain as an iterated integral.
17. Change of variables formula in the integral over a domain $D \subset \mathbf{R}^2$: conditions, formula of the Jacobian, the change of variables formula.
18. Polar coordinates on the plane: definition, examples of domains which are rectangular/normal in polar coordinates. Jacobian and the change of variables formula for polar coordinates.
19. Infinite series: definition of convergence, necessary and sufficient conditions. Examples of convergent and divergent series.
20. Improper integrals of the first and second kinds: definitions, two examples of calculation
21. Comparison criterion for existence of an improper integral: formulation, example. Integral test for absolute convergence of an infinite series: formulation, example.
22. Uniform convergence of functions: definition, three statements on preservation of continuity, integral, and derivative. Two counterexamples for point-wise convergence.
23. Power series: definition, the Cauchy-Hadamard theorem. Radius of convergence: definition, formula. Example of a power series and its radius of convergence.
24. Taylor-Maclaurin series: definition, series for $e^x, \sin x, \cos x, (1+x)^{-1}, \ln(1+x), \operatorname{arctg} x$ and their radii of convergence.