Mathematical Analysis II
Question List for the final Exam.

1. Partial derivatives of a function of several variables: definition, examples.
2. Directional derivatives: definition, relation to partial derivatives.
3. Gradient, its relation to partial and directional derivatives.
4. The formula of small increments
5. The tangent plane to a graph of a function: definition, equation.
6. Higher order partial derivatives: definition, examples.
7. The Schwartz lemma.
8. The Hessian of a function: definition, relation to second derivative of a section of a function in a given direction.
9. Sufficient condition for local convexity/concavity.
10. Positive/negative definiteness of a symmetric matrix: definition and the Sylvester criterium.
11. Local extrema: definition and necessary condition in the terms of the gradient
12. Classification of critical points. Sufficient conditions for a critical point to be a local minimum/local maximum
13. Local extrema under given constraints: definition, the Lagrange multipliers method.
14. The integral of a function of two variables over a rectangle: definition, properties.
15. Normal and regular domains: definitions, examples.
16. Representation of an integral over a normal domain as an iterated integral.
17. Change of variables formula in the integral over a domain $D \subset \mathbf{R}^2$: conditions, formula of the Jacobian, the change of variables formula.
18. Polar coordinates on the plane: definition, examples of domains which are rectangular/normal in polar coordinates. Jacobian and the change of variables formula for polar coordinates.
19. Infinite series: definition of convergence, necessary and sufficient conditions. Examples of convergent and divergent series.
20. Infinite series: definition of convergence and absolute convergence. Example of series which is convergent, but not absolutely convergent.
21. Improper integrals of the first and second kinds: definitions, two examples of calculation
22. Comparison criterion for existence of an improper integral: formulation, example. Integral test for absolute convergence of an infinite series: formulation, example.
23. Uniform convergence of functions: definition, three statements on preservation of continuity, integral, and derivative. Two counterexamples for point-wise convergence.
24. Power series: definition, the Cauchy-Hadamard theorem. Radius of convergence: definition, formula. Example of a power series and its radius of convergence.
25. Taylor-Maclaurin series: definition, series for $e^x, \sin x, \cos x, (1 + x)^{-1}, \ln(1 + x), \operatorname{arctg} x$ and their radii of convergence.
26. Differential equations: definition, reduction of the higher order equations to first order in higher dimension, theorem of existence and uniqueness. Three example of differential equations: (a) solution uniquely exists; (b) solution is not unique (c) solution is unique but exists only up to a 'blow up' moment.
27. Linear systems of differential equations: definition, linear properties of the system of solutions. Fundamental system of solutions: definition, formula in the case when the $n \times n$-matrix of equation has n different eigenvalues.
28. Laplace transform: definition, properties. Example of a function and its Laplace transform. Two formulae for the inverse Laplace transform.
29. Two examples of solving of linear differential equations using Laplace transform.
30. Euler's Gamma and Beta function: definition, properties. The Poisson integral: formula, calculation using the polar coordinates.
31. Fourier transform: definition, formula for the inverse Fourier transform.
32. Fourier series: definition, formula for the coefficients. Example of calculation of the Fourier coefficients.