

Introduction to Mathematical Modelling

Complete Question List.

1. What is the golden ratio?
2. Give three geometric constructions of the golden ratio.
3. What are the golden rectangle and the golden spiral? Draw the picture.
4. What are the golden triangle, the golden gnomon, and the golden pyramid?
5. Draw a pentagram and indicate three pairs of segments in it which constitute the golden ratio.
6. Define the following sets of numbers: natural, integer, rational, real.
7. Is the golden ratio a rational number? Explain the answer.
8. What is the definition of the Fibonacci numbers? What is their relation to the golden ratio?
9. Give the formula for the n -th Fibonacci number.
10. Give the formula for the sum of squares of first n Fibonacci number. What is the Fibonacci spiral? Draw the picture.
11. Formulate the Principle of Mathematical Induction.
12. What is the definition of a general linear regression?
13. What is the characteristic polynomial for a linear regression? What is the characteristic polynomial for Fibonacci numbers? Specify its roots.
14. What is the algorithm to deduce a formula for the n -th element in a linear regression?
15. What is the definition of a Platonic solid? How many Platonic solids exist? Name them.
16. Name all the Platonic solids and indicate which type of faces/vertices each of them have.
17. What is the definition of an Archimedean solid? How many Archimedean solids exist? Give you favorite example of an Archimedean solid, indicate types of its faces/vertices.
18. The Euler formula for polyhedrons.
19. What are the definitions of tessellation, polygonal tessellation? Give examples
20. Define regular and semi-regular polygonal tessellations, give examples.
21. Draw tessellations with the following vertice types: 4-8-8, 3-6-3-6.
22. How many regular polygonal tessellations exist? Draw each of them and indicate its vertice type.
23. The Pick formula for lattice polygons: formulation, examples.
24. Draw two examples of lattice polygons which DO NOT follow the Pick formula. Explain.
25. What is the definition of a convex set? Draw the examples of a convex and non-convex sets
26. What is a convex combination? Draw example of convex combination of (a) tree points; (b) four points.
27. The Radon theorem: general formulation, illustration in \mathfrak{R}^2 .
28. Helly's theorem: formulation for finite number of sets, counter-example for infinite number of sets.
29. Compact sets in \mathfrak{R}^d : definition and the main properties. Examples of compact and non-compact sets.
30. Internal, external, and boundary points for a set: definitions and examples.
31. Helly's theorem for finite number of sets.
32. Star-shaped sets: definition, examples, Krasnosel'skii's theorem.
33. What is the definition of a graph? A complete graph? A bipartite graph?
34. What is a permutation? How many permutations of n -elements set exist?
35. What is the number of m -element subsets of an n -element set?
36. How many edges has the complete graph K_n ? The complete bipartite graph $K_{n,m}$?
37. Planar graphs. The Euler formula for planar graphs.
38. Kuratowski's theorem for planar graphs.
39. The Eulerian paths and the Eulerian circuits. The Eulerian graphs: necessary and sufficient condition.
40. Connected graphs and bridges. The Fleury algorithm.
41. The Hierholtzer algorithm.
42. The Hamiltonian graphs: definition, example.
43. The Dirac theorem. The Ore theorem.
44. Conic sections. Canonic equations for ellipse, hyperbola, and parabola.
45. Foci, directrices, and eccentricity for ellipse, hyperbola, and parabola.