MATHEMATICAL ANALYSIS 2
Problems List 1.
Partial and directional derivatives. Gradient. Tangent plane. Approximate calculations

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1. Calculate partial derivatives of the functions

(a) $f(x,y) = \dfrac{x^2+y^3}{xy^2}$, (b) $f(x,y) = y\sqrt{x^2+y^2}$, (c) $f(x,y) = \cos(x\sin y)$,

(d) $f(x,y) = e^{y^2}\log_2(1+xy)$, (e) $f(x,y) = \ln\left(\sqrt[3]{x^3+y^3}-x\right)$ (f) $f(x,y) = \operatorname{tg}(x\operatorname{arctg}(y))$.

2. Calculate the directional derivatives of the functions in given directions

(a) $f(x,y) = \dfrac{x^2+y}{x^3y^2}, \vec{v} = (1,2)$, (b) $f(x,y) = x\sqrt{x^3+y^3}, \vec{v} = (-1,1)$.

3. Find the direction such that the function $f(x,y) = \sqrt{e^x}(x+y^2)$ at the point $(0,2)$ has the derivative in this direction equal 0.

4. Find the directional derivative of the function $f(x,y) = y - x^2 + 2\ln(xy)$ at the point $(-1/2,-1)$ in the direction $\vec{v}(\alpha)$, which is the unit vector that constitutes the angle α with the positive OX-semiaxis. Find the values of α for which the derivative takes its maximal and minimal values.

5. Calculate the gradients of the functions in given points

(a) $f(x,y) = x^3+xy^2+2,\ (-1,2)$, (b) $f(x,y) = (1+x)^y,\ (1,1)$,

(c) $f(x,y) = \sqrt{e^x}(x+y^2),\ (2,1)$, (d) $f(x,y) = y - x^2 + 2\ln(xy),\ (1,1)$.

6. The altitude $H = 100$ mm and the diameter of the base $D = 50$ mm of a cylinder are measured with the error ± 1 mm. With which accuracy one can give the value for the volume of the cylinder?

7. The lengths of the sides of a rectangular box are measured with the error 5mm each, and the values are $3, 4$, and 5cm. With which accuracy one can give the value

(a) of the volume of the box;

(b) of the surface area of the box?

8. Solve the previous problem if the measurement errors for the sides are $3, 4$, and 5mm respectively.

9. Write the general and the directional forms of equation of the tangent plane to the graph of the function in the given point

(a) $f(x,y) = x^2\sqrt{y^2+1},\ (1,3,z_0)$, (b) $f(x,y) = e^{x+2y},\ (2,-1,z_0)$,

(c) $f(x,y) = \dfrac{\arcsin x}{\arccos x},\ (\dfrac{-1}{2},\dfrac{\sqrt{3}}{2},z_0)$.

10. Find the points on the graph of the function $f(x,y) = \operatorname{arctg}\dfrac{x}{y}$ where the tangent plane is parallel to the plane $x+y-z=5$.

11. Find the tangent plane to the graph of the function $f(x,y) = x^2+y^2$ which is orthogonal to the line $x=t, y=t, z=2t, t\in\mathbf{R}$.