

MATHEMATICAL ANALYSIS 2

Problems List 2.

Second and higher order derivatives. Convexity. Sylvester's criterion.

1. Calculate all partial derivatives of the 2nd order of the functions

$$\begin{aligned} \text{(a)} \quad f(x, y) &= \cos(x^2 + y^2), & \text{(b)} \quad f(x, y) &= ye^{xy}, & \text{(c)} \quad f(x, y) &= x^2 + \frac{y^3}{x}, \\ \text{(d)} \quad f(x, y) &= y \ln \frac{x}{y}, & \text{(e)} \quad f(x, y, z) &= \frac{y}{\sqrt{1+x^2+z^2}} & \text{(f)} \quad f(x, y, z, w) &= \ln(x + y^2 + z^3 + w^4 + 1). \end{aligned}$$

2. Write the Hessian of the function and specify the domains where the Hessian is positive/negatively defined. Investigate the domains of convexity.

$$\begin{aligned} \text{(a)} \quad f(x, y) &= \sin(x^2 + y^2), & \text{(b)} \quad f(x, y) &= xe^{xy}, & \text{(c)} \quad f(x, y) &= x^3 + \frac{y^2}{x}, \\ \text{(d)} \quad f(x, y) &= x \ln \frac{y}{x}, & \text{(e)} \quad f(x, y) &= \frac{y}{\sqrt{1+x^2}} & \text{(f)} \quad f(x, y) &= \ln(x + y^2 + 1). \end{aligned}$$

3. Study the following matrices for being positively/negatively definite or semi-definite.

$$\begin{aligned} \text{(a)} \quad \begin{pmatrix} 2 & 3 \\ 3 & 4 \end{pmatrix}, & \quad \text{(b)} \quad \begin{pmatrix} -1 & 2 \\ 2 & 3 \end{pmatrix}, & \quad \text{(c)} \quad \begin{pmatrix} -1 & 2 & 3 \\ 2 & 3 & 2 \\ 3 & 2 & 1 \end{pmatrix}, \\ \text{(d)} \quad \begin{pmatrix} 2 & 4 & 3 \\ 4 & 5 & 4 \\ 3 & 4 & 12 \end{pmatrix}, & \quad \text{(e)} \quad \begin{pmatrix} 3 & 1 & 5 \\ 1 & 1 & 2 \\ 5 & 2 & 7 \end{pmatrix}, & \quad \text{(f)} \quad \begin{pmatrix} 2 & 2 & 3 & 1 \\ 2 & 5 & 1 & 2 \\ 3 & 1 & 2 & 3 \\ 1 & 2 & 3 & 4 \end{pmatrix}. \end{aligned}$$

4. Find the points of convexity, concavity, and inflection for the function in the given direction.

$$\text{(a)} \quad f(x, y) = x^3 + y^3 - 3x^2y^2, \vec{v} = (-1, 1), \quad \text{(b)} \quad f(x, y) = x^4 + y^4 - 4xy, \vec{v} = (2, -1).$$

5. Find and classify all the critical points of $f(x, y) = 4 + x^3 + y^3 - 3xy$.

6. Find and classify all the critical points of $f(x, y) = 3x^2y + y^3 - 3x^2 - 3y^2 + 5$.

7. Find and classify all the critical points of $f(x, y) = 3xy - \frac{1}{2}y^2 + 2x^3 + \frac{9}{2x^2}$.

8. Calculate all partial derivatives of the 3rd order of the functions

$$\text{(a)} \quad f(x, y) = \cos(x^2 + y^2) \quad \text{(b)} \quad f(x, y) = e^{xy} \quad \text{(c)} \quad f(x, y) = \sqrt{x+y}.$$