MATHEMATICAL ANALYSIS 2

Problems List 3.

Local and global extrema. Conditional extrema: Lagrange's multipliers method.

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1. Given the objective function $f(x,y) = x^2 + y^2$, subject to the constraint $g(x,y) = x^2 + y^2 - 4x - 2y - 15$ find extremal points.

2. A rectangular box that is open at the top must have a volume of 32 cm^3 What must its dimensions be so that its total area will be minimal?

3. Determine the point on the plane $4x - 2y + z = 1$ that is closest to the point $(-2, -1, 5)$.

4. On the surface $2x^2 + 3y^2 + 3z^2 - 12xy + 4xz = 85$ find the points of maximal and minimal values for z.

5. Find the global extrema of $f(x,y) = x^2 + 4y^2$ on the domain bounded by the curves $x^2 + (y+1)^2 = 4, y = -1$, and $y = x + 1$.

6. Find the global extrema of $f(x,y) = x^2 + y^2 - 6x + 6y$ on the disk of radius 2, centred at the origin.

7. Find the maximal and the minimal values of the functions on the given domains:

(a) $f(x,y) = 2x^3 + 4x^2 + y^2 - 2xy$, $D = \{(x,y) : x^2 \leqslant y \leqslant 4\}$;

(b) $f(x,y) = \sqrt{y - x^2} + \sqrt{x - y^2}$, $D = \{(x,y) : y \geqslant x^2, x \geqslant y^2\}$;

(c) $f(x,y) = \sqrt{1 - x^2} + \sqrt{4 - x^2 - y^2}$, $D = \{(x,y) : x^2 \leqslant 1, x^2 + y^2 \leqslant 4\}$;

(d) $f(x,y) = x^2 - y^2$, D is the triangle with the vertices $(0,1), (0,2), (1,2)$;

(e) $f(x,y) = x^4 + y^4$, $D = \{(x,y) : x^2 + y^2 \leqslant 9\}$

8. Krzysztof and Szymon consume two products. If x and y are the quantities of the products (respectively), then Krzysztof's utility function is $U(x,y) = \ln x + 2\ln y$, and Szymon's is $U(x,y) = xy^2$. The prices of the products per unit are: $P_x = 5$ and $P_y = 2$. Krzysztof and Szymon have identical incomes, of 90 złeach. Find the optimal consumed quantity from each of the two products, for each of Krzysztof and Szymon.