MATHEMATICAL ANALYSIS 2
Problems List 4.
Definite integrals of one and two variables. Double and iterated integrals

1. Calculate the definite integral. Draw a picture.

(a) $\int_{-1}^{1} e^{-x}\, dx$,　(b) $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \mathrm{ctg} x\, dx$,　(c) $\int_{e^{-2}}^{e^2} \ln x\, dx$. (d) $\int_{0}^{\pi} \sin 2x\, dx$,

2. Calculate the definite integral over the given rectangles.

(a) $\iint_R (x^2 + y^3 - xy)\, dxdy$, $R = [0,1] \times [0,1]$,　(b) $\iint_R \dfrac{x}{y^2}\, dxdy$, $R = [1,2] \times [2,4]$,

(c) $\iint_R (1 + x + y)^3\, dxdy$, $R = [0,2] \times [0,1]$,　(d) $\iint_R x\sin(xy)\, dxdy$, $R = [0,1] \times [\pi, 2\pi]$,

(e) $\iint_R \dfrac{x+y}{e^x}\, dxdy$, $R = [0,1] \times [0,1]$,　(f) $\iint_R e^{2x-y}\, dxdy$, $R = [0,1] \times [-1,0]$.

3. Calculate the iterated integrals and draw the domains of integration

(a) $\int_1^2 dx \int_x^{x^2} \dfrac{x}{y}\, dy$,　(b) $\int_0^1 dx \int_x^{x^2} \dfrac{y}{x^2}\, dy$,　(c) $\int_1^4 dx \int_x^{2x} x^2\sqrt{y-x}\, dy$,　(d) $\int_0^3 dx \int_0^x \sqrt{x^2 + 16}\, dy$.

4. Change the order of integration in the iterated integrals and calculate the integrals

(a) $\int_1^2 dx \int_x^{x^2} \dfrac{x}{y}\, dy$,　(b) $\int_1^4 dx \int_x^{3x} x\sqrt{y-x}\, dy$,　(c) $\int_0^3 dx \int_0^x \sqrt{x^2 + 1}\, dy$, (d) $\int_1^e dx \int_{\ln x}^1 \dfrac{1}{e^y - 1}\, dy$.

5. Calculate the integrals over the normal domains bounded by the given curves

(a) $\iint_D xy^2 dxdy$, $y = x$, $y = 2 - x^2$,　(a) $\iint_D x^2 y\, dxdy$, $y = -2$, $y = \dfrac{1}{x}$, $y = -\sqrt{-x}$,

(c) $\iint_D e^{x/y} dxdy$, $y = \sqrt{x}$, $x = 0$, $y = 1$,　(d) $\iint_D (xy + 4x^2) dxdy$, $y = x + 3$, $y = x^2 + 3x + 3$.