MATHEMATICAL ANALYSIS 2

Problems List 5.

Change of variables formula for an integral of two variables. Polar coordinates.

---

1. Determine the domain Δ which is transformed by the given mapping to the given domain D.

(a) D is bounded by $y = -x+4$, $y = x+1$, and $y = \dfrac{x-4}{3}$, the transformation $x = \dfrac{1}{2}(u+v), y = \dfrac{1}{2}(u-v)$

(b) D is the ellipse $x^2 + \dfrac{y^2}{36} \leqslant 1$, the transformation $x = \dfrac{u}{2}, y = 3v$.

(c) D is the parallelogram with the vertices $(1,0), (4,3), (1,6)$ and $(-2,3)$, the transformation $x = \dfrac{1}{2}(u+v), y = \dfrac{1}{2}(u-v)$.

(d) D is the parallelogram with vertices $(2,0), (5,3), (6,7)$ and $(3,4)$, the transformation $x = \dfrac{1}{3}(v-u), y = \dfrac{1}{3}(4v-u)$.

(e) D is the domain bounded by $xy = 1, xy = 3, y = 2$ and $y = 6$, the transformation $x = \dfrac{v}{6u}, y = 2u$.

2. Derive a transformation that will represent the triangle D with vertices $(1,0), (6,0)$ and $(3,8)$ as an image of a right triangle with the right angle occurring at the origin of the u, v system.

3. Calculate the Jacobians of the transformations:

(a) $x = 4u - 3v^2 \qquad y = u^2 - 6v$;

(b) $x = u^2v^3 \qquad y = 4 - 2\sqrt{u}$;

(c) $x = \dfrac{v}{u} \qquad y = u^2 - 4v^2$.

4. Perform the change of variables to the polar coordinates and evaluate the integrals. Draw the domain of integration in the Cartesian and polar coordinates

(a) $\iint_D xy\,dxdy$, $D : x^2 + y^2 \leqslant 1, \dfrac{x}{\sqrt{3}} \leqslant y \leqslant x\sqrt{3}$;

(b) $\iint_D y^2 e^{x^2+y^2}\,dxdy$, $D : x^2 + y^2 \leqslant 1, x \geqslant 0, y \geqslant 0$;

(c) $\iint\limits_D (y^2 + 3x)\,dxdy$, D is the region in the 3rd quadrant between $x^2 + y^2 = 1$ and $x^2 + y^2 = 9$;

(d) $\iint\limits_D (4xy - 7)\,dxdy$, D is the portion of $x^2 + y^2 \leqslant 2$ in the 1st quadrant.

5. Performing an appropriate change of variables, evaluate the integrals

(a) $\iint\limits_D 6x - 3y\,dxdy$ where R is the parallelogram with vertices $(2,0), (5,3), (6,7)$ and $(3,4)$ **Hint:** compare with Problem 1(d);

(b) $\iint\limits_D xy^3\,dxdy$ where D is the domain bounded by $xy = 1, xy = 3, y = 2$ and $y = 6$. **Hint:** compare with Problem 1(e);

(c) $\iint\limits_D x + 2y \, dxdy$ where D is the triangle with vertices $(0,3), (4,1)$ and $(2,6)$ **Hint:** proceed similarly to Problem 2;

(d) $\iint\limits_D x^2 \, dxdy$, where D is the ellipse $x^2 + \dfrac{y^2}{36} \leqslant 1$ **Hint:** use Problem 1(b) and then change coordinates to polar;

6. Find the area of the ellipse $(x-3)^2 + 4(y+1)^2 \leqslant 10$.