MATHEMATICAL ANALYSIS 2

Problems List 6.

Power series. Taylor-Maclaurin series.

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1. Determine the Taylor-Maclaurin series for the given function at the point $x_0 = 0$

(a) $f(x) = \cos(4x)$;

(b) $f(x) = x^6 e^{2x^3}$;

(c) $f(x) = x^2 \cos 2x^3$;

(d) $f(x) = \dfrac{x^{100}}{1+x^3}$.

2. For each function from the previous problem find $f^{(2019)}(0)$.

3. Determine the Taylor-Maclaurin series for the function $(1+x)^a$ at the point $x_0 = 0$ (the *Generailized Binomial Formula*).

4. Using the Generalized Binomial Formula, determine the Taylor-Maclaurin series for the given function at the point $x_0 = 0$

(a) $f(x) = \sqrt{1-x^2}$;

(b) $f(x) = \dfrac{1}{\sqrt[3]{1+x^3}}$;

(c) $f(x) = \dfrac{x^3}{\sqrt{x^2+16}}$;

(d) $f(x) = \dfrac{x^{100}}{1+x^3}$.

5. Using the Taylor-Maclaurin series and differentiation/integration calculate the infinite sums

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n3^n}$;

(b) $\displaystyle\sum_{n=2}^{\infty} \frac{2^n - 1}{3^n}$;

(c) $\displaystyle\sum_{n=0}^{\infty} \frac{n(n+1)}{5^n}$;

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{(n+1)2^n}$;

(e)* $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n(n+2)}$ (*Hint:* consider the limit of $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{n(n+2)}$ as $x \nearrow 1$).

6. Determine the Taylor-Maclaurin series for the given function $f(x)$ and x_0. Provide **two** solutions: using the formula for the coefficients and the change of variables.

(a) $f(x) = e^{-6x}, x_0 = -4$;

(b) $f(x) = \ln(3+4x), x_0 = 1$;

(c) $f(x) = \dfrac{7}{x^4}, x_0 = -3$;

7. For each of the series in the previous problem determine the interval of convergence.