Mathematical Analysis II
Question List for the final Exam.

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1. Partial derivatives of a function of several variables: definition, examples.
2. Directional derivatives: definition, relation to partial derivatives.
3. Gradient, its relation to partial and directional derivatives.
4. The formula of small increments
5. The tangent plane to a graph of a function: definition, equation.
6. Higher order partial derivatives: definition, examples.
7. The Schwartz lemma.
8. The Hessian of a function: definition, relation to second derivative of a section of a function in a given direction.
9. Sufficient condition for local convexity/concavity.
10. Positive/negative definiteness of a symmetric matrix: definition and the Sylvester criterium.
11. Local extrema: definition and necessary condition in the terms of the gradient
12. Classification of critical points. Sufficient conditions for a critical point to be a local minimum/local maximum
13. Local extrema under given constraints: definition, the Lagrange multipliers method.
14. The double integral of a function of two variables over a rectangle: definition, properties.
15. Normal and regular domains: definitions, examples.
16. Representation of a double integral over a normal domain as an iterated integral.
17. Change of variables formula in the integral over a domain $D \subset \mathbf{R}^2$: conditions, formula of the Jacobian, the change of variables formula.
18. Polar coordinates on the plane: definition, examples of domains which are rectangular/normal in polar coordinates. Jacobian and the change of variables formula for polar coordinates.
19. Cylindrical and spherical coordinates in $\mathbf{R}^3$: definition, relation to the Carthesian coordinates. Jacobian and the change of variables formula for polar coordinates.
20. Application of double and triple integrals in mechanics and geometry.
21. Differential equations: definition, reduction of the higher order equations to first order in higher dimension, theorem of existence and uniqueness. Three example of differential equations: (a) solution uniquely exists; (b) solution is not unique (c) solution is unique but exists only up to a 'blow up' moment.
22. Special classes of ODEs and methods of their solutions: separable, Bernoulli, homogeneous.
23. Linear systems of differential equations: definition, linear properties of the system of solutions. Fundamental system of solutions: definition, formula in the case when the $n \times n$-matrix of equation has n different eigenvalues.
24. Non-homogeneous linear systems of ODEs: definition, examples, method of variation of the unknown constant.
25. Laplace transform: definition, properties. Example of a function and its Laplace transform. Two formulae for the inverse Laplace transform.
26. Two examples of solving of linear differential equations using Laplace transform.
27. Difference equations. The analogue of the Euler method for solving linear difference equations.
28. Generating functions: definition, properties, examples. Example of solving a linear difference equation using generating functions.
29. Partial differential equations (PDEs): order of an equation, linear, semi-linear and quasi-linear PDEs.
30. The characteristic method for 1st order PDEs: description, example of application.
31. Classification of the 2nd order linear PDEs. Examples.
32. The characteristic method for 2st order hyperbolic PDEs: description, example of application.
33. The separation of variables (Fourier) method: description, example of application.