

# MATHEMATICAL ANALYSIS 2

## Test 2, version A.

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1. Calculate the double integral

$$\iint_D \frac{1}{y\sqrt{y-x}} dx dy, \quad D = \{(x, y) : 0 \leq x \leq y \leq 1\}.$$

Draw the domain of integration.

2. Write the change of variables formula for double integrals. Perform the change of variables  $x = 1 + \rho \cos \phi$ ,  $y = -1 + \frac{1}{2} \sin \phi$  and calculate the integral

$$\iint_D xy dx dy, \quad D = \{(x, y) : (x-1)^2 + 4(y+1)^2 \leq 2\}.$$

Draw the domain of integration on  $(x, y)$ -plane and  $(\rho, \phi)$ -plane.

3. Write the change of variables formula to the polar coordinates. Changing coordinates to polar, calculate

$$\iint_D (x^2 + xy) dx dy, \quad D = \{(x, y) : x \geq y, x \geq -\sqrt{3}y\}.$$

Draw the domain of integration in  $(x, y)$ - and polar coordinates.

4. Write the formulae for the static moments, the center of mass, and moments of inertia for a material plate  $D$  with the density function  $\gamma(x, y)$ . Calculate the moments of inertia of the quarter of a circle of radius  $D$ , located in the 1st quadrant, with the density function  $\gamma(x, y) = x^2$ .