

MATHEMATICAL ANALYSIS 2

Test 2, version B.

1. Calculate the double integral

$$\iint_D \frac{1}{y\sqrt{y-x}} dx dy, \quad D = \{(x, y) : 0 \leq x \leq y \leq 1\}.$$

Draw the domain of integration.

2. Write the change of variables formula for double integrals. Perform the change of variables $x = u^{2/3}v^{-2/3}$, $y = u^{4/3}v^{-1/3}$ and calculate the integral

$$\iint_D xy dx dy, \quad D = \{(x, y) : \sqrt{x} \leq y \leq 2\sqrt{x}, x^2 \leq y \leq 2x^2\}.$$

Draw the domain of integration on (x, y) -plane and (u, v) -plane.

3. Write the change of variables formula to the polar coordinates. Changing coordinates to polar, calculate

$$\iint_D (xy - y^2) dx dy, \quad D = \{(x, y) : x \leq y, x \geq -\sqrt{3}y\}.$$

Draw the domain of integration in (x, y) - and polar coordinates.

4. Write the formulae for the static moments, the center of mass, and moments of inertia for a material plate D with the density function $\gamma(x, y)$. Calculate the moments of inertia of the plate

$$D = \{(x, y) : 0 \leq x \leq 2y \leq 1\}$$

with the density function $\gamma(x, y) = 1 - x$.