MATHEMATICAL ANALYSIS 2

Test 2, version C.

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1. Calculate the double integral

$$\iint_D \frac{y}{\sqrt{y+x}}\,dxdy, \quad D=\{(x,y): 0\leqslant x\leqslant y\leqslant 1\}.$$

Draw the domain of integration.

2. Write the change of variables formula for double integrals. Perform the change of variables $x=-1+\rho\cos\phi, y=1+\frac{1}{3}\sin\phi$ and calculate the integral

$$\iint_D (x^2+y^2)\,dxdy, \quad D=\{(x,y): (x+1)^2+9(y-1)^2\leqslant 3\}.$$

Draw the domain of integration on (x,y)-plane and (ρ,ϕ)-plane.

3. Write the change of variables formula to the polar coordinates. Changing coordinates to polar, calculate

$$\iint_D (xy+y^2)\ dxdxy, \quad D=\{(x,y): x\leqslant y, x\leqslant -\sqrt{3}y\}.$$

Draw the domain of integration in (x,y)- and polar coordinates.

4. Write the formulae for the static moments, the center of mass, and moments of inertia for a material plate D with the density function $\gamma(x,y)$. Calculate the moments of inertia of the quarter of a circle of radius D, located in the 3rd quadrant, with the density function $\gamma(x,y)=xy$.