

MATHEMATICAL ANALYSIS 2

Test 2, version D.

1. Calculate the double integral

$$\iint_D \frac{\sqrt{y+x}}{y} dx dy, \quad D = \{(x, y) : 0 \leq x \leq y \leq 1\}.$$

Draw the domain of integration.

2. Write the change of variables formula for double integrals. Perform the change of variables $x = u^{-1/3}v^{1/3}$, $y = u^{1/3}v^{2/3}$ and calculate the integral

$$\iint_D xy dx dy, \quad D = \{(x, y) : x \geq 0, x^2 \leq y \leq 2x^2, x^{-1} \leq y \leq 2x^{-1}\}.$$

Draw the domain of integration on (x, y) -plane and (u, v) -plane.

3. Write the change of variables formula to the polar coordinates. Changing coordinates to polar, calculate

$$\iint_D (x + y - xy) dx dy, \quad D = \{(x, y) : x \leq y, \sqrt{3}x \geq y\}.$$

Draw the domain of integration in (x, y) - and polar coordinates.

4. Write the formulae for the static moments, the center of mass, and moments of inertia for a material plate D with the density function $\gamma(x, y)$. Calculate the moments of inertia of the plate

$$D = \{(x, y) : 0 \leq 2x \leq 3y \leq 1\}$$

with the density function $\gamma(x, y) = y$.