MATHEMATICAL ANALYSIS 2

Final Test, version A.

1. (1p.) Write the general and the directional forms of equation of the tangent plane to the graph of the function $z = \frac{\ln x}{\sqrt{y^2 - 1}}$ at the point $(1, 3, z_0)$.

2. (2p.) Which points are called critical? What types of critical points there exist? Find and classify all the critical points of the function $f(x, y) = x^2 + y^2 - 3xy + 3x + 5y$.

3. (3p.) Write the formulae for moments of inertia for a material body U with the density function $\gamma(x, y, z)$. Calculate the moments of inertia of the cylinder $U = \{(x, y, z) : |z| \leq 1, x^2 + y^2 \leq 4\}$ with the density function $\gamma(x, y, z) = z^2$.

4. (3p.) Write the change of variables formula in a double integral. Performing a proper change of variables, calculate

$$\iint_{D} (x-y)^2 \, dx dxy, \quad D = \{(x,y) : y^2 \leqslant x \leqslant 2y^2, x^2 \leqslant y \leqslant 2y^2\}.$$

Draw the domain of integration in (x, y)- and new coordinates.