MATHEMATICAL ANALYSIS 2

Final Test, version C.

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1. (1p.) Write the general and the directional forms of equation of the tangent plane to the graph of the function $z = \dfrac{\cos(xy)}{e^{x^2-y^2}}$ at the point $(-\pi, \pi, z_0)$.

2. (2p.) Which points are called critical? What types of critical points there exist? Find and classify all the critical points of the function $f(x, y) = x^2 + 2y^2 + 3xy + 4x + 5y$.

3. (3p.) Write the formulae for moments of inertia for a material body U with the density function $\gamma(x, y, z)$. Calculate the moments of inertia of the cone $U = \{(x, y, z) : x^2 + y^2 \leqslant z^2 \leqslant 1\}$ with the density function $\gamma(x, y, z) = z^2$.

4. (3p.) Write the change of variables formula in a double integral. Performing a proper change of variables, calculate

$$\iint_D (x - y)^2 \, dxdxy, \quad D = \{(x, y) : 1 \leqslant xy \leqslant 2, y^2 \leqslant x \leqslant 2y^2\}.$$

Draw the domain of integration in (x, y)- and new coordinates.