MATHEMATICAL ANALYSIS 2
Final Test, version D.

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1. (1p.) Write the general and the directional forms of equation of the tangent plane to the graph of the function $z = \dfrac{\cos(x^2 + y^2)}{e^{xy}}$ at the point $(-\pi, \pi, z_0)$.

2. (2p.) Which points are called critical? What types of critical points there exist? Find and classify all the critical points of the function $f(x, y) = x^2 + y^2 - 4xy + x - y$.

3. (3p.) Write the formulae for moments of inertia for a material body U with the density function $\gamma(x, y, z)$. Calculate the moments of inertia of the cylindrical ring $U = \{(x, y, z) : 1 \leqslant x^2 + y^2 \leqslant 2, |z| \leqslant 1\}$ with the density function $\gamma(x, y, z) = z^2$.

4. (3p.) Write the change of variables formula in a double integral. Performing a proper change of variables, calculate

$$\iint_D (x^3 + y^3)\, dxdxy, \quad D = \{(x, y) : 1 \leqslant xy \leqslant 2, y^3 \leqslant x \leqslant 2y^3\}.$$

Draw the domain of integration in (x, y)- and new coordinates.