ALGEBRA
Exam

1. (4 points) Find the complex solutions to the equation $z^2 + 2iz + i - 1 = 0$. The answer give in the Carthesian form.

2. (5 points) Decompose the rational function $\dfrac{x^3 + 1}{x^4 + x^2 + 1}$ into irreducible real fractions.

3. (4 points) Tetrahedron T has a vertex at the origin O and other three vertices P_1, P_2, P_3 obtained by intersection of the lines

$$\ell_1 : \begin{cases} x = 2t \\ y = -t \\ z = t \end{cases} , \quad \ell_2 : \begin{cases} x = -t \\ y = t \\ z = 2t \end{cases} , \quad \ell_3 : \begin{cases} x = t \\ y = 2t \\ z = -t \end{cases} ,$$

with the plane

$$x + y + z = 1.$$

Find the volume of the tetrahedron T.

4. (4 points) Find the inverse matrix to $\begin{pmatrix} 1 & -2 & 3 \\ 2 & -3 & 4 \\ 3 & -4 & 6 \end{pmatrix}$.

5. (4 points) Solve the system of linear equations

$$\begin{cases} x + 2y - 3z + v = 1 \\ -2x - 3y + 4z - v = 2 \\ 3x + 8y - 12z + 2v = -1 \end{cases} .$$

6. (5 points) Find eigenvalues and eigenvectors of the matrix $\begin{pmatrix} 3 & 2 & -1 \\ -4 & -3 & 2 \\ -14 & -7 & 6 \end{pmatrix}$.