# ALGEBRA
## Exam

**1.** (4 points) Find the complex solutions to the equation $z^2 - iz - z - i = 0$. The answer give in the Carthesian form.

**2.** (5 points) Decompose the rational function $\dfrac{x^3 - 2}{x^4 + 2x^2 - 3}$ into irreducible real fractions.

**3.** (4 points) Tetrahedron $T$ has a vertex at the origin $O$ and other three vertices $P_1, P_2, P_3$ obtained by intersection of the lines

$$\ell_1 : \begin{cases} x = t \\ y = 2t \\ z = 3t \end{cases}, \quad \ell_2 : \begin{cases} x = 3t \\ y = 2t \\ z = t \end{cases}, \quad \ell_3 : \begin{cases} x = t \\ y = 3t \\ z = 2t \end{cases},$$

with the plane

$$x + y + z = 1.$$

Find the volume of the tetrahedron $T$.

**4.** (4 points) Find the inverse matrix to $\begin{pmatrix} 1 & -2 & 3 \\ 2 & -3 & 4 \\ 3 & -4 & 6 \end{pmatrix}$.

**5.** (4 points) Solve the system of linear equations

$$\begin{cases} x + 2y - 3z + v = 1 \\ -2x - 3y + 4z - v = 2 \\ 3x + 8y - 12z + 2v = -1 \end{cases}.$$

**6.** (5 points) Find eigenvalues and eigenvectors of the matrix $\begin{pmatrix} 7 & 9 & 3 \\ -2 & -2 & -1 \\ 4 & 2 & 1 \end{pmatrix}$.