

# ALGEBRA

## Exam

1. (4 points) Find the complex solutions to the equation  $z^2 - 2iz - i + 1 = 0$ . The answer give in the Carthesian form.

2. (5 points) Decompose the rational function  $\frac{x^3 - 1}{x^4 - x^2 + 1}$  into irreducible real fractions.

3. (4 points) Tetrahedron  $T$  has a vertex at the origin  $O$  and other three vertices  $P_1, P_2, P_3$  obtained by intersection of the lines

$$\ell_1 : \begin{cases} x = t \\ y = -t \\ z = 2t \end{cases}, \quad \ell_2 : \begin{cases} x = -2t \\ y = t \\ z = -t \end{cases}, \quad \ell_3 : \begin{cases} x = t \\ y = -2t \\ z = -t \end{cases},$$

with the plane

$$x + y + z = 1.$$

Find the volume of the tetrahedron  $T$ .

4. (4 points) Find the inverse matrix to  $\begin{pmatrix} 1 & 2 & -3 \\ -2 & -3 & 4 \\ 3 & 8 & -12 \end{pmatrix}$ .

5. (4 points) Solve the system of linear equations

$$\begin{cases} x - 2y + 3z + v = 1 \\ 2x - 3y + 4z - 2v = -1 \\ 3x - 4y + 6z + 3v = -2 \end{cases}.$$

6. (5 points) Find eigenvalues and eigenvectors of the matrix  $\begin{pmatrix} -1 & 2 & -1 \\ 4 & -3 & 2 \\ 12 & -6 & 5 \end{pmatrix}$ .