

# ALGEBRA

## Exam

1. (4 points) Find the complex solutions to the equation  $z^2 + iz + z - 1 = 0$ . The answer give in the Carthesian form.
2. (5 points) Decompose the rational function  $\frac{x^3 + 1}{x^4 - x^2 - 2}$  into irreducible real fractions.
3. (4 points) Tetrahedron  $T$  has a vertex at the origin  $O$  and other three vertices  $P_1, P_2, P_3$  obtained by intersection of the lines

$$\ell_1 : \begin{cases} x = t \\ y = t \\ z = 3t \end{cases}, \quad \ell_2 : \begin{cases} x = 3t \\ y = t \\ z = t \end{cases}, \quad \ell_3 : \begin{cases} x = t \\ y = 3t \\ z = t \end{cases},$$

with the plane

$$x + y + z = 1.$$

Find the volume of the tetrahedron  $T$ .

4. (4 points) Find the inverse matrix to  $\begin{pmatrix} 1 & 2 & 1 \\ -1 & -1 & -2 \\ 1 & 4 & 0 \end{pmatrix}$ .
5. (4 points) Solve the system of linear equations

$$\begin{cases} x + 2y - 3z + v = 2 \\ -2x - 3y + 7z - v = 1 \\ 3x + 5y - 9z + v = 1 \end{cases}.$$

6. (5 points) Find eigenvalues and eigenvectors of the matrix  $\begin{pmatrix} -14 & 3 & 6 \\ 5 & 0 & -2 \\ -10 & -2 & 3 \end{pmatrix}$ .