ALGEBRA
Exam

1. (4 points) Find the complex solutions to the equation $z^2 - iz - z - i = 0$. The answer give in the Carthesian form.

2. (5 points) Decompose the rational function $\dfrac{x^3 - 2}{x^4 + 2x^2 - 3}$ into irreducible real fractions.

3. (4 points) Tetrahedron T has a vertex at the origin O and other three vertices P_1, P_2, P_3 obtained by intersection of the lines

$$\ell_1 : \begin{cases} x = t \\ y = 2t \\ z = 3t \end{cases}, \quad \ell_2 : \begin{cases} x = 3t \\ y = 2t \\ z = t \end{cases}, \quad \ell_3 : \begin{cases} x = t \\ y = 3t \\ z = 2t \end{cases},$$

with the plane

$$x + y + z = 1.$$

Find the volume of the tetrahedron T.

4. (4 points) Find the inverse matrix to $\begin{pmatrix} 1 & 1 & -2 \\ -1 & 0 & 3 \\ 2 & 1 & -4 \end{pmatrix}$.

5. (4 points) Solve the system of linear equations

$$\begin{cases} x + 2y + z - 2v = 1 \\ -x - y - 2z + v = -1 \\ x + 4y - v = 0 \end{cases}.$$

6. (5 points) Find eigenvalues and eigenvectors of the matrix $\begin{pmatrix} 7 & 9 & 3 \\ -2 & -2 & -1 \\ 4 & 2 & 1 \end{pmatrix}$.