ALGEBRA
Exam

1. (4 points) Find the complex solutions to the equation $z^2 + 2iz + i - 1 = 0$. The answer give in the Carthesian form.

2. (5 points) Decompose the rational function $\dfrac{x^3+1}{x^4+x^2+1}$ into irreducible real fractions.

3. (4 points) Tetrahedron T has a vertex at the origin O and other three vertices P_1, P_2, P_3 obtained by intersection of the lines

$$\ell_1 : \begin{cases} x = 2t \\ y = -t \\ z = t \end{cases}, \quad \ell_2 : \begin{cases} x = -t \\ y = t \\ z = 2t \end{cases}, \quad \ell_3 : \begin{cases} x = t \\ y = 2t \\ z = -t \end{cases},$$

with the plane

$$x + y + z = 1.$$

Find the volume of the tetrahedron T.

4. (4 points) Find the inverse matrix to $\begin{pmatrix} 1 & 2 & -3 \\ -2 & -3 & 4 \\ 3 & 8 & -12 \end{pmatrix}$.

5. (4 points) Solve the system of linear equations

$$\begin{cases} x - 2y + 3z + v = 1 \\ 2x - 3y + 4z - 2v = -1 \\ 3x - 4y + 6z + 3v = -2 \end{cases}.$$

6. (5 points) Find eigenvalues and eigenvectors of the matrix $\begin{pmatrix} 3 & 2 & -1 \\ -4 & -3 & 2 \\ -14 & -7 & 6 \end{pmatrix}$.