ALGEBRA
Exam

1. (4 points) Find the complex solutions to the equation $z^2 + iz + z - 1 = 0$. The answer give in the Carthesian form.

2. (5 points) Decompose the rational function $\dfrac{x^3+1}{x^4-x^2-2}$ into irreducible real fractions.

3. (4 points) Tetrahedron T has a vertex at the origin O and other three vertices P_1, P_2, P_3 obtained by intersection of the lines

$$\ell_1 : \begin{cases} x = t \\ y = t \\ z = 3t \end{cases}, \quad \ell_2 : \begin{cases} x = 3t \\ y = t \\ z = t \end{cases}, \quad \ell_3 : \begin{cases} x = t \\ y = 3t \\ z = t \end{cases},$$

with the plane

$$x + y + z = 1.$$

Find the volume of the tetrahedron T.

4. (4 points) Find the inverse matrix to $\begin{pmatrix} 1 & 2 & -3 \\ -2 & -3 & 7 \\ 3 & 5 & -9 \end{pmatrix}$.

5. (4 points) Solve the system of linear equations

$$\begin{cases} x + y - 2z + v = 2 \\ -x + 3z - v = 1 \\ x + y - 4z + 2v = -1 \end{cases}.$$

6. (5 points) Find eigenvalues and eigenvectors of the matrix $\begin{pmatrix} -14 & 3 & 6 \\ 5 & 0 & -2 \\ -10 & -2 & 3 \end{pmatrix}$.