

MATHEMATICAL ANALYSIS 2

Problems List 1.

Partial and directional derivatives. Gradient. Tangent plane. Approximate calculations

1. Calculate partial derivatives of the functions

$$\begin{aligned} \text{(a)} \quad f(x, y) &= \frac{x^2 + y^3}{xy^2}, & \text{(b)} \quad f(x, y) &= y\sqrt{x^2 + y^2}, & \text{(c)} \quad f(x, y) &= \cos(x \sin y), \\ \text{(d)} \quad f(x, y) &= e^{y^2} \log_2(1 + xy), & \text{(e)} \quad f(x, y) &= \ln\left(\sqrt[3]{x^3 + y^3} - x\right) & \text{(f)} \quad f(x, y) &= \operatorname{tg}(x \operatorname{arctg}(y)). \end{aligned}$$

2. Calculate the directional derivatives of the functions in given directions

$$\text{(a)} \quad f(x, y) = \frac{x^2 + y}{x^3 y^2}, \vec{v} = (1, 2), \quad \text{(b)} \quad f(x, y) = x\sqrt{x^3 + y^3}, \vec{v} = (-1, 1).$$

3. Calculate the gradients of the functions in given points

$$\begin{aligned} \text{(a)} \quad f(x, y) &= x^3 + xy^2 + 2, (-1, 2), & \text{(b)} \quad f(x, y) &= (1 + x)^y, (1, 1), \\ \text{(c)} \quad f(x, y) &= \sqrt{e^x(x + y^2)}, (2, 1), & \text{(d)} \quad f(x, y) &= y - x^2 + 2 \ln(xy), (1, 1). \end{aligned}$$

4. Find the unit vector such that the function $f(x, y) = \sqrt{e^x(x + y^2)}$ at the point $(0, 2)$ has the derivative in this direction equal 0. Draw the picture.

5. Find the directional derivative of the function $f(x, y) = y - x^2 + 2 \ln(xy)$ at the point $(-1/2, -1)$ in the direction $\vec{v}(\alpha)$, which is the unit vector that constitutes the angle α with the positive OX -semiaxis. Find the values of α for which the derivative takes its maximal and minimal values. Draw the picture.

6. The altitude $H = 100$ mm and the diameter of the base $D = 50$ mm of a cylinder are measured with the error ± 1 mm. With which accuracy one can give the value for the volume of the cylinder?

7. The lengths of the sides of a rectangular box are measured with the error 5mm each, and the values are 3, 4, and 5cm. With which accuracy one can give the value

(a) of the volume of the box;

(b) of the surface area of the box?

8. Solve the previous problem if the measurement errors for the sides are 3, 4, and 5mm respectively.

9. Write the general and the directional forms of equation of the tangent plane to the graph of the function in the given point

$$\begin{aligned} \text{(a)} \quad f(x, y) &= x^2 \sqrt{y^3 + x^2}, (1, 2, z_0), & \text{(b)} \quad f(x, y) &= e^{x+2y}, (2, -1, z_0), \\ \text{(c)} \quad f(x, y) &= \frac{\arcsin x}{\arccos x}, \left(\frac{-1}{2}, \frac{\sqrt{3}}{2}, z_0\right). \end{aligned}$$

10. Find the points on the graph of the function $f(x, y) = \ln\left(\frac{x+y}{y}\right)$ where the tangent plane is parallel to the plane $x + 2y - 3z = 0$.

11. Find the tangent plane to the graph $f(x, y) = e^{\frac{x-y}{y}}$ where the tangent plane is parallel to the plane $x - y - 2z = 0$.

12. Find the tangent plane to the graph of the function $f(x, y) = x^2 + y^2$ which is orthogonal to the line $x = t, y = t, z = 2t, t \in \mathbf{R}$.