

ALGEBRA
Final test

1. (4 points) Decompose the rational function $\frac{x^2 + x + 1}{x^3 + x^2 + x + 1}$ into irreducible real fractions.
2. (3 points) Find normal and parametric equations of the plane which contains the points $P = (1, -1, 2)$, $Q = (1, 3, 5)$ and $S = (3, 0, 1)$.

3. (3 points) Do the vectors

$$(1, 0, 1), \quad (0, 1, 1), \quad (1, 1, 0)$$

form a basis in \mathbb{R}^3 ? If yes, find the coordinates of the vector $(1, 2, 3)$ in this basis

4. (3 points) Calculate the determinant $\begin{vmatrix} 1 & 0 & -1 & 1 \\ -2 & 1 & 0 & 3 \\ 2 & 3 & 1 & -2 \\ 0 & 1 & -1 & 0 \end{vmatrix}$.

5. (4 points) Find complex eigenvalues and eigenvectors of the matrix

$$\begin{pmatrix} 2 & 1 \\ -1 & 3 \end{pmatrix}.$$